

UNESCO-IHE INSTITUTE FOR WATER EDUCATION



Modelling and Optimization for Groundwater Resources Development: Case study of Akaki Catchment Well fields in Addis Ababa, Ethiopia

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**MODELLING AND OPTIMIZATION FOR
GROUNDWATER RESOURCES DEVELOPMENT:
CASE STUDY OF AKAKI CATCHMENT WELL FIELDS
IN ADDIS ABABA, ETHIOPIA**

Master of Science Thesis

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The findings, interpretations and conclusions expressed in this study do neither necessarily reflect the views of the UNESCO-IHE Institute for Water Education, nor of the individual members of the MSc committee, nor of their respective employers.

Dedicated to my son Eyoel, my wife Hirut and to my parents!

Abstract

Groundwater resource is main source of water supply for many countries in the world. Likewise groundwater is used as source of potable water in the city of Addis Ababa, the capital of Ethiopia, in addition to surface water resources. The abstraction of groundwater resource in well fields of Akaki catchment is currently not based on understanding of drawdown levels and available potential groundwater resources in the area. Meanwhile, this mismanagement of groundwater resource and over exploitation in the Akaki catchment has caused continuous decline of groundwater level. The objective of this study is to find the maximum groundwater drawdown and optimal abstraction rates for each well within the well fields.

Different techniques for developing groundwater management models (coupled simulation-optimization approaches) are proposed by a number of researchers, which enable determination of optimal abstraction rates of wells given certain constraints. In this study the groundwater management model (MODMAN) which links the groundwater simulation model (MODFLOW) with the optimization model (LINDO) is used to find optimal abstraction rates and well locations. For this study a regional single layer groundwater model of Akaki catchment developed in 2000E.C by BCEOM in cooperation with SEURECA and Tropics consulting Engineers Plc (later, in 2004 E.C- adjusted by enlarging the model span) is used as groundwater simulation model. This regional model was adjusted with additional grid refinement in the area of interest addressed in this study.

To determine the optimal abstraction rates and well spacing- two objectives are formulated for both steady and transient state condition. These are maximization of abstraction rate and minimization of operational, drilling and pipe costs. To achieve these objectives minimum and maximum drawdown constraints of 15 and 30 m are imposed at twenty three control locations. Balance constraints are also imposed for cost minimization. Linear optimization and mixed-integer programming techniques are used for formulating and solving the optimization problems. The research led to new insights on specification of reduced number of drawdown constraints, depending on transmissivity values of the aquifer.

The optimal abstraction rate of selected wells for 30 years varies from ~ 50,000 m³/day in first period, ~40,000 m³/day in second stress period and ~43,000 m³/day in third stress period keeping the maximum drawdown from 15 m to 25 m.

It is also concluded that the optimal abstraction rates do not cause significant depletion of the groundwater resources in the area of the well fields or significant decrease of groundwater outflow to nearby rivers, springs and downstream groundwater flow further away from the well fields.

The study strongly recommends further improvement of the regional groundwater model of Akaki catchment by using multilayer approach, with defined geometry of aquifers, and sub-sequent re-assessment of the optimization results. It is also recommended to compare the results from the implemented simulation-optimization approach with coupling of simulation model with other global optimization algorithms (e.g. genetic algorithms).

Key words: MODMAN, MODFLOW, LINDO, Optimization, well fields and Akaki

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List of symbols

MODMAN	MODflow MANagemnet
MODFLOW	Modular Three Dimensional Ground Water Flow Model
LINDO	Linear Interactive Discrete Optimizer
E.C	Ethiopian calendar
AAWSSA	Addis Ababa Water Supply and Sewerage Authority
ANN	Artificial Neural Network
LP	Linear Programming
MIP	Mixed Integer Programming
GA	Genetic Algorithm
MNN	Modular Neural Network
HS	Harmony Search
EA	Evolutionary Algorithm
3-D	Three Dimensional
CSA	Central statistical Agency
USGS	United State Geological Survey
MPS	Mathematical Programming System
ETB	Ethiopian Birr

1 INTRODUCTION

1.1 BACKGROUND

Both surface and groundwater resources are water supply source of Addis Ababa city, the capital of Ethiopia. In recent years, the population of city is alarmingly increasing. This continuous increase of population has caused high demand of potable water supply in the city. However, the existing water supply sources of the city do not have sufficient capacity to satisfy this increasing demand of potable water. In order to satisfy the demand, alternative sources of water are investigated by Addis Ababa Water Supply and Sewerage Authority (AAWSSA). Therefore, AAWSSA has identified potential groundwater sources as additional sources of water supply for the city. Since the cost for treatment of surface water is too high, surface water sources are not included in current plans for additional water supply sources. Groundwater source in the area does not require any treatment as compared to surface water sources since the aquifer in the area is deep enough and does not have any contact with contaminants.

The potential ground water resources of the city are found within Akaki catchment in several existing and potential well fields, namely Akaki, Fanta, Dalota, and Dukem up and Dukem down. Among these well fields Akaki well field is both existing and potential source of water supply for the city. The rest of the well fields are identified as potential well fields to be used in long run in addition to the existing Akaki well field. The ground water model was developed for these well fields for management purposes.

The ground water model of Akaki catchment was developed in 2000 E.C by BCEOM in cooperation with SEURECA and Tropics consulting Engineers Plc (later, in 2004- adjusted by enlarging the model span). The developed model is used as ground water management tool for the identified well fields. In this model set- up aquifer is considered as single layer because of lack of knowledge about the complex geological structure in the area. The thickness of the aquifer is set at a constant value of 100 m while its transmissivity is varying in space. With this simplification the aquifer in the area is considered as confined even though unconfined aquifers exist in large parts of the well field areas.

Despite the presence of developed regional groundwater model of Akaki catchment, there is no good management of groundwater in the well fields. The common problem in the area is high abstraction of groundwater for meeting high demand of potable water in the city. Particularly in the Akaki well field high draw down of ground water table due to over exploitation is observed, and this is identified as the main ground water management problem. While the aquifer is assessed to be highly productive, this high drawdown may lead to increased well installation and operation costs if future expansion plans are implemented.

This study aims at addressing the issues of ground water table decline in the well fields by finding optimal ground water abstraction rates and well locations. In order to determine the sustainable yield from the aquifer system and the capacity of each well field, two objective functions are formulated and drawdown constraints of 15, 20, 25, and 30 m are used.

1.2 PROBLEM STATEMENT

Akaki aquifer system serves as one of the potential source of groundwater to provide potable water supply for the city of Addis Ababa in addition to surface water sources. The area is characterized by Tertiary volcanic rocks covered with thick residual and alluvial soils (Ayenew, et al., 2008). Currently, approximately 25 percent of the water supply to the city comes from groundwater, particularly from the Akaki well field located in the southern part of Akaki catchment Demlie 2007 (cited by Ayenew, et al. (2008)). More than 100 boreholes are already constructed with in the Akaki catchment, many of which are domestic wells with low capacity. Among these, 26 high capacity wells are located within the Akaki well field. The depth of boreholes drilled in this well field, which are used for water supply of the city via the distribution system ranges from 119 to 170 m.

It is clear that understanding abstraction of well rates with available potential of groundwater resource enables to overcome the overexploitation of groundwater resource. In order to plan and manage potential groundwater within Akaki catchment it is important to understand the behaviour of hydrologic systems within the Akaki catchment using groundwater model. Even though regional groundwater model was developed for Akaki catchment well fields in 2000 and modified in 2004 by AG consultant and AAWSA (Ayenew, et al. ((2008))), management of groundwater in the well fields is still characterized with large uncertainties. Nevertheless, based on the groundwater model developed, a prediction of pumping rates from Akaki well field was proposed ((Tesfaye, 2009) with a recommendation for continuous monitoring of the pumping rate and drawdown of the water table.

Water table decline (groundwater drawdown) is a result of overexploitation. These changes in water table map potentially affect surface water and other ecosystems in the area. Due to continuously increasing number of population in the city and expansion of industries near to the well field, high abstraction of groundwater is common in the area. Often groundwater abstractions are carried out without the basic understanding of the groundwater recharge, lateral and vertical extent of the aquifers, and the available groundwater reserve in the area (Ayenew, et al., 2008). With this approach problems of different nature are occurring. Firstly, the intensive pumping of groundwater from the well field results in decline of groundwater level which potentially facilitates the flow of water from contaminated Akaki River to the shallow aquifer within the Akaki catchment. Secondly, in the Akaki well field itself, which captures water from the deep aquifer, significant draw downs may lead to increased future installation and operational

costs. Over the last three years the water level in the well field declined by an average of about 0.15 m per year (Tesfaye, 2009).

One additional problem that is not addressed in this study, but needs to be mentioned is the fact that in Addis Ababa city there is only one treatment plant which is situated at Kaliti. The capacity of the treatment plant is much less than produced sewerage of city. In addition the constructed sewerage system does not cover all parts of the city. Some of the sewerage is transported to the treatment plant by truck in addition to the part conveyed by the sewerage line system. However, most sewerage from residential houses, hotels, industries, hospitals and farmlands is disposed directly in to Akaki River prior to treatment. This disposed sewerage highly polluted the river and it completely changed the door of the river. Since surface and groundwater need to be treated as integrated resources due to their interactions with the polluted surface water, there is also probability that the groundwater may also be polluted. This in turn may result in big investment costs to clean the groundwater and even it may be difficult to clean it. This situation may be of importance for groundwater found in the shallow aquifer within the broader Akaki catchment.

The deep aquifer within the Akaki catchment well fields, receives recharge from a combination of sources: infiltration from precipitation, which takes place over a wider area than the well fields, infiltration from existing reservoirs in the catchment, and possibly from the neighbouring aquifer systems. With the operation of the existing wells, there is already a significant decline of water tables in the well field. Therefore, in order to overcome the problem of groundwater table decline within the well fields, developing groundwater management model for optimal groundwater abstraction and well location is proposed as potential solution.

1.3 GENERAL OBJECTIVE

The main objective of the study is to determine optimal abstraction rates and well locations in the Akaki catchment wellfields namely Akaki, Fanta, Dalota, Dukem up and Dukem down that minimize the groundwater drawdown in the well fields.

1.4 SPECIFIC OBJECTIVE

Listed below are the specific objectives of this thesis:

- To set minimal groundwater table drawdown, which will be acceptable for future exploitation of the well fields
- To find optimal abstraction rates and locations of wells in the well fields, in terms of maximizing the total abstraction from the well fields, with respect to the acceptable draw downs

-
- To find the optimal abstraction rates and well locations in terms of minimizing of installation and operational costs, for the assessed maximum abstraction rates, with respect to the acceptable draw downs
 - To compare results from steady and transient simulation-optimization formulations, with respect to the objectives listed above

1.5 RESEARCH QUESTIONS

Following the problem description and the specific objectives the following research questions are formulated for this study:

1. What is the most appropriate way of specifying drawdown constraints for solving the optimization problems identified (maximization total abstraction or minimization of costs)?
2. What will be the optimal abstraction rates and locations of the wells in the well fields (related to the two formulations of maximization of total abstraction and to minimization of the costs of their installation and operation) that keep minimal groundwater table drawdown in the well fields? Which optimal solution from the two formulations is to be recommended?
3. What is the difference between the steady and transient simulation-optimization solutions, and which solution should be recommended?

1.6 OVERVIEW OF STUDY AREA

1.6.1 Location

The Akaki catchment is located in the central Ethiopian highlands at the western edge of the Main Ethiopian Rift (MER). The total surface area of the catchment is 1600 km². It is bounded between 8°45' 20" to 9°13' 17" N latitude and 38°34' 3" to 39°4'10" E longitude (Ayenew, et al., 2008). The Akaki well field is located to the southeast of Akaki town about 22 km south of the centre of Addis Ababa within the Akaki catchment whereas the rest of the well fields are located near to Akaki well field.

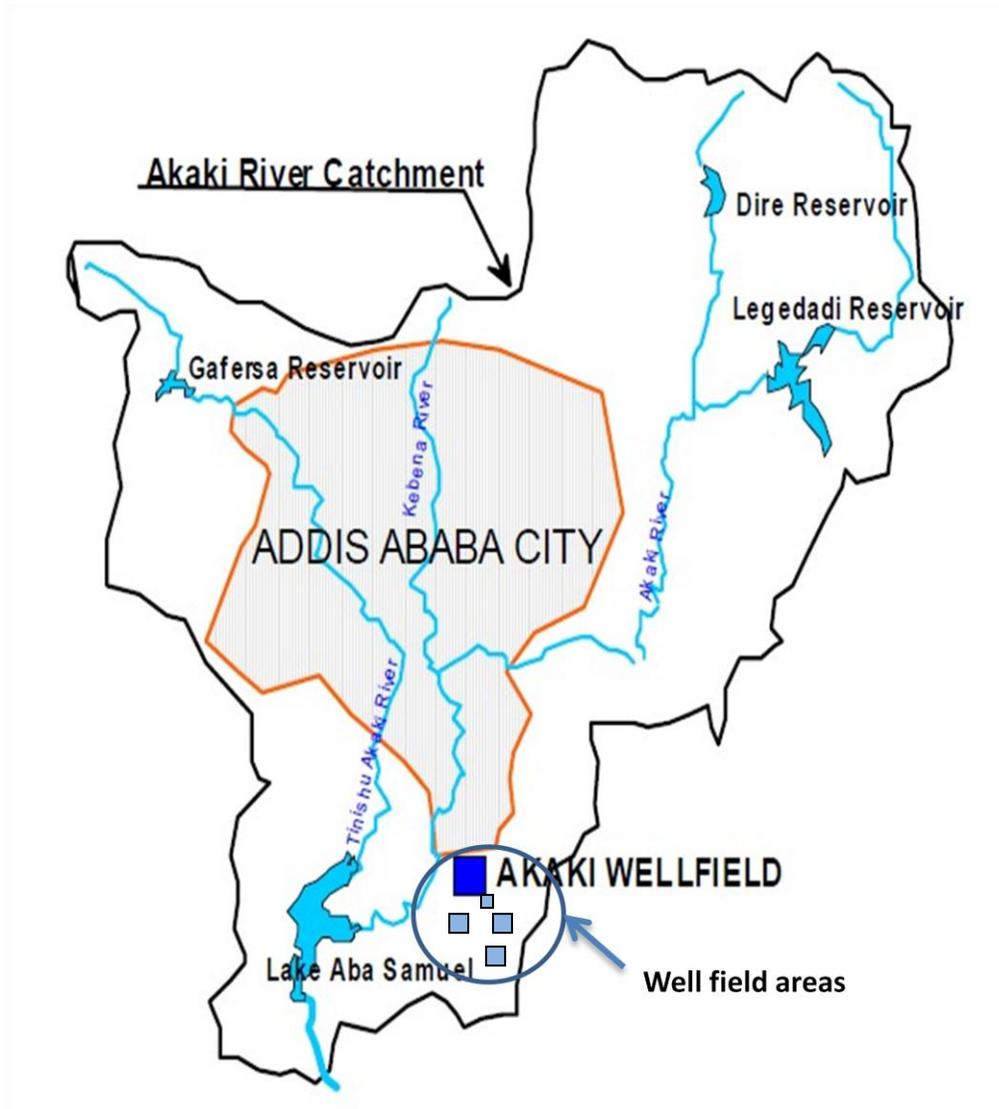


Figure 1-1 Location of study area (Tsehayu, et al., 2002)

1.6.2 Administration and Population

Addis Ababa is capital city of Ethiopia and found within the Akaki catchment. The population of Addis Ababa city is 3,627,934 as of 2007 Central Statistical Agency (CSA) report.

1.6.3 Land use

Forests, urban area, agricultural and open areas are common land features of Akaki catchment(BCEOM, 2000 and 2002). Forests are commonly found in the upper part of the catchment particularly in northern part of the catchment. The urban area is mainly paved surface area (with the designed and partly implemented designed drainage system of the Addis Ababa

city) and agricultural areas are found in large part of the catchment especially in east, south and south west.

1.6.4 Physiography

The Akaki River originates from the Entoto Mountain and joins Awash River 95km away from the source. The drainage system of the Akaki River covers catchment area of 1600km². Within the catchment there are perennial rivers such as small and big Akaki Rivers and Kebena River. Three surface reservoirs (Legedadi, Dire and Gefersa) at the upstream part of the catchment are constructed. These are used for the domestic water supply of the city, while non functional (due to over siltation) Aba Samuel reservoir also exists at downstream part of catchment.

1.6.5 Geology

Volcanic rocks of different age are predominantly found in Akaki River catchment, Addis Ababa city as well as in its surroundings. According to the report of(BCEOM, 2000 and 2002), different types of rocks exist in various parts of the catchment. These are summarized as follows: In Entoto Mountain, northern and north-eastern Addis Ababa: trachytes, rhyolites and basalts are commonly found. Around western and south eastern parts of Addis Ababa; younger volcanic rocks, rhyolites, ignimbrites, trachytes and trachybasalts are predominant. Lacustrine deposits, alluvial and residual soils are also common between Abasamuel, Akaki town and small Akaki River and also between Dukem and Debreziet towns. In addition, around Akaki well field area olivine basalts, scoria, vascular basalt and scoriaceous basalt are predominately found.

1.6.6 Hydrogeology

In the catchment area of Akaki volcanic rocks, weathered and fractured rocks are most common. They are formed due to tectonic effect. Most of these rocks have faults, fractures, and joints. The aquifer of the area is mostly unconfined aquifer and due to complex geology in the area, it is difficult to build its geometry. The thickness of the aquifer is estimated at(BCEOM, 2000 and 2002). According to(Tesfaye, 2009), the aquifers with in Akaki catchment are classified as below:

- Scoria, scoriaceous basalt and inter-formational gravel and sand layers constitute highly productive aquifers with primarily porosity and permeability
- Highly weathered and fractured basalts, fractured tuffs, ignimbrite and other pyroclastics constitute highly productive aquifers of secondary porosity and permeability
- Basalt with some fractures, vesicles and sparsely spaced joints, ignimbrite and agglomerates form moderately productive aquifers in the area

As a result, from the combination of different geology, porosity and permeability of the area; the aquifer of Akaki catchment are classified as shallow (along the river valley), deep (well field area) and thermal aquifer (located at larger depth beyond 300m) (Tesfaye, 2009).

1.6.7 Hydrology

Akaki catchment has extensive drainage system mainly composed of Big Akaki and Small Akaki River. The two rivers meet at the manmade reservoir called Aba Samuel reservoir. The Akaki River is gauged at Akaki Bridge and flows to Awash River. The mean annual flow of the river at this gauging station is 339mm(Tesfaye, 2009). By using semi- distributed water balance model, the recharge of the catchment is assessed(BCEOM, 2000 and 2002). During this assessment of recharge, in order to keep the spatial variation the catchment area is divided in to two parts. The upper part of the catchment, which is mostly urban area, has low recharge, whereas the lower part of the catchment has higher recharge. The recharge value of the upper part is 33 mm/year and the lower part of the catchment recharge value is 74 mm/year(BCEOM, 2000 and 2002).

1.6.8 Soil types and permeability

Alluvial soils, residual soils and lacustrine soils are common soil types within the Akaki catchment(BCEOM, 2000 and 2002). The alluvial soils are found mainly in middle to lower reaches of the river; residual soils are common in the upper part of the catchment, whereas the lacustrine soils which are black cotton soils are common in southern and south-eastern part of the area.

According to (BCEOM, 2000 and 2002) site tests were carried out to determine the permeability of the soil in the area. The investigation showed that in most of the lacustrine soil there is no infiltration of rain to the ground. Accordingly, the permeability of the catchment is classified as low, lower and medium. Especially, in the Akaki well field the permeability of the soil is higher than the rest of areas within the catchment.

2 LITERATURE REVIEW

2.1 Introduction

Interactions between surface and subsurface water are the main parts of the hydrologic balance on the catchment scale. According to (Thomas .W. C, et al., 1998) almost all surface-water features such as streams, lakes, reservoirs, wetlands, and estuaries interact with the sub surface water system at any time. The main causes for the existence of interaction between surface and groundwater are lateral flow through the unsaturated zone and infiltration or ex-filtration from the saturated flow(Sophocleous, 2002). Depending on the climatic condition, hydraulic head and hydraulic conductivity the interaction between them may be gaining or losing.

As the interaction between surface and groundwater exists, both resources are used for different purposes to fill the ultimate demand of human being. However, demand for fresh water will increase as population increases despite limited sources of fresh water on and below the earth surface. In most countries ground water is used as potable source of drinking water. During the utilization of groundwater for water supply, overexploitation of groundwater may occur leading to ground water table decline which in turn results in the following problems (David P.Ahlfeld and E.Mulligan, 2000): 1) subsidence of the overlaying geological strata, 2) saltwater intrusion in to fresh water, 3) groundwater quality degradation 4) high installation and operational cost of pumping wells.

In order to deal with the aforementioned problems, groundwater management has to be carried out throughout the whole period of ground water utilization. For management of groundwater and decision making, groundwater simulation models are commonly used. These models are capable to show the response of groundwater systems to human interference (David P.Ahlfeld and E.Mulligan, 2000).Groundwater management modelling, however, needs combination of management(e.g. optimization) and simulation models; former provides desired operational values and later provides the aquifer situation in which, at the end, optimal water use will be provided (Lall and Santini, 1989). Depending on the nature of the management problem, the groundwater table conditions and aquifer properties, different optimization algorithms have been developed and applied such as linear, mixed integer, genetic, and dynamic algorithms.

Management of groundwater as a scarce resource is associated with determination of appropriate abstraction alternatives and treatment of aquifers as storage systems within complex environment. These activities are supported by formulation of groundwater management problems as mathematical/ optimization problems(Schwarz, 1976).Optimization algorithms are commonly used to determine the optimal abstraction rates, locations and drawdown of wells. This is commonly done by treating the well rates and/or locations as decision variables, while introducing a number of additional constraints on abstraction rates and groundwater heads, or associated variables, such as balance constraints, velocity, or gradient constraints. A

management problem can then be posed by formulating certain objective function (minimization or maximization of abstraction rates, cost minimization, head minimization at certain locations, etc.) This objective function can be linear and non linear. This depends on the formulation of management problem and on the type of aquifer that is being considered (confined or unconfined). In order to have solution for this optimization problem(linear or non linear), linkage between simulation and optimization model has to be done via a management model (Psilovikos, 1999). Approaches for linking simulation and optimization models are presented in the following section.

2.2 Methodology used

Different approaches are proposed by different researchers to solve groundwater management problems by linking simulation and optimization models. The review of this linking simulation and optimization approaches are summarized by Gorelick (1983), Yeh(1992), Ahlfeld and Heidari (1994), and Das and Datta (2001). These approaches are categorized as embedding, unit response, global algorithms (with advance of fast computers), and emulators (surrogates) algorithms (expensive computation). Each of this approaches are presented below.

The embedding approach for solving groundwater management problems was first developed by Aguado and Remson (1974).This approach incorporates the governing partial differential equation for groundwater flow as a constraint in an optimization model for aquifer management (Tung and Koltermann, 1985). As discussed in paper of Tung and Koltermann (1985), mostly this approach was applied in small-scale problems and do not come across computational problems. However, they tried to look at the computational aspects of the embedding approach during large scale groundwater management problems. Especially for complex unsteady state simulation models this approach becomes difficult for implementation.

The unit response approach works on the principle of superposition and it is mostly applicable when the aquifer system is linear or approximately linear, and the boundary conditions are homogeneous (Das and Datta, 2001). However, for highly nonlinear aquifer systems the application of response matrix is not good enough (Rosenwald and Green, 1974). Jonoski, et al.(1997) used response matrix approach for optimization of artificial recharge-pumping systems to provide maximum abstraction rate through artificial recharge; Wattenberger (1970) used a transient response matrix to develop linear programming to maximize well production; Deninger (1970), used non equilibrium formula of Theis (1935) to obtain response matrix for maximization of water production from well fields; Atwood and Gorelick (1985) also used response matrix approach for removing groundwater contaminants.

Another approach which became quite popular with the advance of fast computers is global optimization. In this approach, simulation model which uses finite difference groundwater equation is combined with a global optimization algorithm (very often genetic algorithm (GA) is used) to determine the optimal groundwater abstraction rates. Masky, et al.(2002) applied global

optimization in groundwater remediation strategy and planning; Tamer Ayvaz and Karahan (2008) also applied this approach in identification of well location and optimal abstraction rate of wells in two dimensional aquifer system; Mirghani, et al.(2009) applied an evolutionary algorithm (EA) for groundwater source identification; Ritzel, et al (1994) used genetic algorithms to solve a multiple objective groundwater pollution containment problem; Tamer Ayvaz (2009) used harmony search (HS) algorithm to find solution for groundwater management problems; McKinney and Lin (1994) also used genetic algorithm to solve groundwater management problem.

Lastly, when the groundwater simulation model is complex (especially for unsteady state) the global optimization approach may become computationally expensive. Therefore, some approaches such as including simpler emulators (surrogates) of the simulation model in the global optimization are introduced to overcome these problems. Kourakos and Mantoglou (2009) used evolutionary algorithms and surrogate modular neural network models in optimization of pumping of coastal aquifers; Sreekanth and Datta (2010) applied genetic algorithm with modular neural network (MNN) as surrogate model in multi objective management of saltwater intrusion in coastal aquifers; Mcphee and Yeh (2008) used model reduction via empirical orthogonal functions for groundwater management problem; Rogers and Dowla (1994) used artificial neural networks with parallel solute transport modelling for optimization of groundwater remediation; Maskey, et al. (2000) also used groundwater model approximation with artificial neural network for selecting optimum pumping strategy for plume removal.

In this study, from the above mentioned approaches for linking simulation and optimization models, the response matrix approach is used to determine the abstraction rates and well locations in Akaki catchment well field areas. This is because the aquifer system of Akaki catchment is considered as confined, i.e., linear. The software package MODMAN is used to generate the response matrix through simulation of MODFLOW and then, the optimization problem is solved by LINDO. Therefore, the methods and tools used in this study are presented in more detail in the following sections.

2.2.1 Simulation Model

Groundwater simulation models are used to provide detailed groundwater heads and flow distributions of complex aquifer systems in a given problem area. As explained in the previous section, these simulation models are then linked with different optimization algorithm to obtain optimal solution, e.g. groundwater abstraction rate and well spacing.

The very common simulation model used for groundwater modelling is MODFLOW. It is 3-D finite difference method for modelling groundwater flow. It solves the groundwater flow partial differential equation, which describes the three dimensional movement of groundwater of constant density through porous media. This basic three dimensional differential equation of groundwater movement is given below (McDonald and Harbaugh, 1988).

$$\frac{\partial}{\partial x} \left(K_{xx} \cdot \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_{yy} \cdot \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_{zz} \cdot \frac{\partial h}{\partial z} \right) - W = S_s \cdot \frac{\partial h}{\partial t}$$

Where

K_{xx}, K_{yy}, K_{zz} are x, y, z coordinate hydraulic conductivity value parallel to the major axes of hydraulic conductivity [LT^{-1}]

h groundwater head [L]

W volumetric flux per unit volume may be terms of sources or sinks of water [T^{-1}]

S_s specific storage of the porous material L^{-1}

t time [T]

This finite difference groundwater flow equation is solved by iterative numerical methods. Different boundary and initial conditions are also required to solve the equation.

2.2.2 Management Model

As explained in previous section, MODMAN is groundwater management model which provides linkage between MODFLOW and optimization tool called LINDO (Psilovikos, 1999). MODMAN uses response matrix technique to transform groundwater development problem into a linear or mixed-integer program (Greenwald, 1998). The response matrix for groundwater head is based on linear space superposition for steady state flow; and both space and time superposition for transient flow. The linear superposition has two principles: 1) multiplication of a well rate by a factor increases drawdown induced by that well by same factor; 2) Drawdown induced by more than one well is equal to the sum of drawdown induced by each individual well. It is summarised mathematically below (Greenwald, 1998, Psilovikos, 1999).

For steady state condition:

$$H_i = U_i - \sum_{j=1}^N \sigma_{ij} \cdot Q_j \quad 2-1$$

For Transient state condition:

$$H_i^T = U_i^T - \sum_{k=1}^T \sum_{j=1}^N \alpha_{ij}^{T-(k-1)} \cdot Q_j^k \quad 2-2$$

Where:

U_i unmanaged head at control location i

Q_j pumping rate at well j

α_{ij} average drawdown in each i observation well to a unit rate pumping at j managed wells

U_i^T unmanaged head at control point i at the end of last managing period t .

H_i^T managed head at control point i at the end of last managing period t .

$\alpha_{ij}^{T-(k-1)}$ average drawdown in each i observation well at the end of the T pumping period due to a unit rate of pumping at the j managed well applied throughout the k pumping period (Colarullo, S.M.Heidari, T.Maddock III, 1984 as cited by (Psilovikos, 1999).

Q_j^k pumping rate at well j during the k pumping period.

The linear drawdown response by each of the j number of wells is obtained from a simulation model which is run with a unit abstraction rate for each of these wells. The unit responses can then be summed to obtain α_{ij} in the above equations and the final equations for managed head become available for formulation of the groundwater management problem as a linear optimization problem that can be solved by linear programming.

2.2.3 Optimization Model

2.2.3.1 Linear programming: (Greenwald, 1998)

Linear programming is defined as a set of decision variables, an objective function and constraints. The objective function is mathematical representation of quantity to be minimized or maximized. Linear programming in groundwater management system is applicable following the linear response theory which uses the principle of linear superposition, described in previous section. In case of linear programming, linear objective function that needs to satisfy all constraints are also formulated as follows (Psilovikos, 1999).

For steady state condition:

$$Z = \sum_{j=1}^N C_j \cdot Q_j \quad 2-3$$

For unsteady state condition

$$Z = \sum_{k=1}^T \sum_{j=1}^N C_j^k \cdot Q_j^k \quad 2-4$$

C_j^k and C_j are cost coefficients. For quantitative management they are 1 but for total cost management they may represent costs coefficient.

From the linear responses introduced earlier, linear constraints can be formulated in terms of draw downs at specific control locations.

For steady state condition:

$$\Delta h_i = U_i - H_i = \sum_{j=1}^N \sigma_{ij} \cdot Q_j \leq b_i = U_i - H_{i,min} \quad 2-5$$

For unsteady state condition:

$$\Delta h_i^T = U_i^T - H_i^T = \sum_{k=1}^T \sum_{j=1}^N \alpha_{ij}^{T-(k-1)} \cdot Q_j^k \leq b_i^T = U_i^T - H_{i,min}^T \quad 2-6$$

Other linear constraints can similarly be formulated (in terms of heads, heads differences, velocity or gradient constraints).

Balance constraints can also be formulated to the total quantity of abstraction of water from some or all of the managed wells. In groundwater resources such constraints may be associated with provision of minimum quantities required to meet the water demand.

For steady state condition:

$$\sum_{j=1}^N Q_j = ct(\text{constant}) \quad 2-7$$

Unsteady state condition

$$\sum_{k=1}^T \sum_{j=1}^N Q_j^k = ct(\text{constant}) \quad 2-8$$

Lastly, constraints are also set for minimum and maximum abstraction rates of each managed wells at each control location.

$$0 < Q_j^k < Q_j^k \text{max} \quad 2-9$$

Where:

$i = 1, \dots, N$ Control location of managed wells

$j = 1, \dots, N$ managed well

$k = 1, \dots, T$ managing period

ct represents constant value

b_i maximum allowable head at control point i

$H_{i,min}$ minimum allowable head at control i

b_i^T maximum managed drawdown at control point i at the end of last stress period T

$H_{i,min}^T$ minimum allowable head at control point i at the end of last stress period T

This approach is mostly applicable when the response of the aquifer due to different stresses is linear. Linear programming is commonly used in confined aquifers since groundwater head in confined aquifer is linearly proportional to hydraulic conductivity and aquifer physical parameters. If nonlinearities are small the same approach can be used for unconfined also.

2.2.3.2 *Mixed Integer programming*

According to (Greenwald, 1998, Psilovikos, 1999), integer mixed programming is an extension of linear programming with constraints that allow for choosing K active wells among J potential wells. This selection is done by using well on/off binary integer variable constraint and integer variable summation constraints. The former binary variables are introduced as follows; if the well is on, the binary value has a value of 1, if the well is not on the value is zero. In the later case, an integer summation variable limits the total number of active wells.

In case of on/off constraint the form will be(Greenwald, 1998):

$$Q_j + M \cdot I_j \geq 0 \quad \text{Extraction of well} \quad 2-10$$

$$Q_j - M \cdot I_j \leq 0 \quad \text{Injection of wells} \quad 2-11$$

Where:

Q_j stress rate at well j (negative for pumping)

M a large positive number with an absolute value greater than that of largest well rate.

I_j a binary variable acting as on/off switch for well j .

Whereas in case of integer variable summation constraint, the form will be:

$$\sum_{j=1}^J I_j \leq K, j = 1, \dots, J \quad 2-12$$

Where:

K is the number of required wells

J potential well sites

I_j binary variables

Linear programming problems are solved by standard algorithms based on the Simplex method and the mixed integer problems extend with the branch and bound algorithms. These are implemented in optimization packages such as LINDO, which will be used in this thesis.

2.3 Previous works of Akaki catchment

A groundwater model is already developed for Akaki catchment in 2000 E.C by BCEOM in cooperation with SEURECA and Tropics consulting Engineers Plc (later, in 2004- adjusted by enlarging the model span). Based on developed model a prediction of sustainable pumping rate from well field was proposed with a continuous monitoring of the pumping rate and drawdown. Ayenew, et al.(2008) also tried to quantify the groundwater fluxes and to analyze the subsurface hydrodynamics in Akaki catchment by giving particular emphasis to the well field that supplies water to city of Addis Ababa using a study state MODFLOW model. Tsehayu, et al (2002) also studied the developed model prediction with the monitored results up on groundwater level in Akaki well field. The model result shows that it is possible to pump from Akaki well field 30,000 m³/day to 35,000 m³/day water but at the end of 20 years pumping will cause 20 to 23 m drawdown in Akaki town (BCEOM, 2000 and 2002).

2.4 Regional groundwater model of Akaki catchment (BCEOM, 2000 and 2002)

The following considerations were taken into account for the development of regional groundwater model of Akaki catchment.

- Groundwater, springs and rivers are being recharged from precipitation taking place within Akaki catchment. Akaki River catchment can be considered as one hydrologic unit.
- The groundwater head map is continuous from north (Entoto area) to south towards well fields generally following the topographic gradient.
- The occurrence of groundwater at the well field is due to hydrologic and hydro geological conditions within Akaki catchment and well field area. Therefore, the potential of well field is directly influenced by the recharge of model area and the conditions in the well field.
- Beyond the well field areas, the groundwater flows towards the south-southeast (Dukem plain) crossing Akaki river catchment.
- The available data for the model area shows that the geological conditions of the area is very complex and using of multiple layers for the model is impossible.
- Given the hydrological and hydro geological condition of the area, the well field must be modelled by considering the whole Akaki catchment area.

2.4.1 Model set up

The regional groundwater model of Akaki catchment was set up by using Processing MODFLOW (Version 5.0.54) software. The model area encompasses the regional groundwater flow system in Akaki catchment, from the river sources located in north and to south it extends to Awash River and Debreziet town (see again, Figure 1-1). The northern, western and eastern

catchment boundaries of Akaki regional groundwater are considered as no flow boundary conditions. Constant head boundary was used in between Dukem Awash and Debreziet depending on groundwater head obtained from the borehole data. The model area is 2254km². The model grid consists of 106 columns and 136 rows. The spacing of the grid is variable in X and Y directions, starting from 1000 m and then gradually reducing to 500 m and 250 m in the central area of the model where the Akaki well field is located. The model is developed as single layer aquifer with variable transmissivity and constant thickness of 100 m. It was impossible to build the real geometry of the aquifer, as a result of insufficient data about the complex geology of the area.

Mostly the recharge of the aquifer system of the study comes from the infiltration of rain. Semi distributed water balance model at monthly time step was developed to determine the recharge of model area. The obtained result was an average recharge of 51mm per year. However, in order to keep the spatial distribution of recharge in the model area, according to hydro geological conditions of the area two recharge zones were considered. These are: 1) areas with high runoff especially in northern part, near to mountains have recharge of 33 mm/year; 2) for the rest of the area a recharge value of 74 mm/year is used in the regional model. In addition to natural recharge from infiltration, the MODFLOW well package is used to specify small amount of additional recharge by leakage from the three reservoirs in the catchment.

The regional groundwater model is also composed of groundwater outputs such as springs (Fanta, Akaki gorge), rivers and pumping wells. The MODFLOW river package is used for specification of the main rivers, the well package for the wells in the well field, and the drain package for simulating the springs in the area.

2.4.2 Model calibration

Firstly, the model was calibrated in steady state condition. This enabled justification for selected assumptions of the modelling and identifying the transmissivity of aquifer. Transmissivity values obtained from borehole tests were used as model start. Then, the transmissivity values are adjusted until the model output is similar to the observed groundwater head surface and observed discharge of Akaki River, Fanta and Aba Samuel gorge springs(BCEOM, 2000 and 2002).

Secondly, the transient model calibration was carried out. It was done by including time variation in the model and storage coefficient of aquifer. Time series of groundwater head of some wells and flows of springs, as well as storage coefficients of test pumping wells are used during the calibration. Based on the observed groundwater head the storage coefficient has been calibrated (BCEOM, 2000 and 2002).

As shown in figure2-1 below the transmissivity of the aquifer varies throughout the model area. High transmissivity value of 0.25m²/s is found near to well fields. The storage coefficient of the well field is high with value of 20% and it varies from 0.5% to 4% throughout the model area.

And also figure 2-2 shows the hydraulic head distribution of the regional model and the flow of groundwater is from North to south of the catchment area.

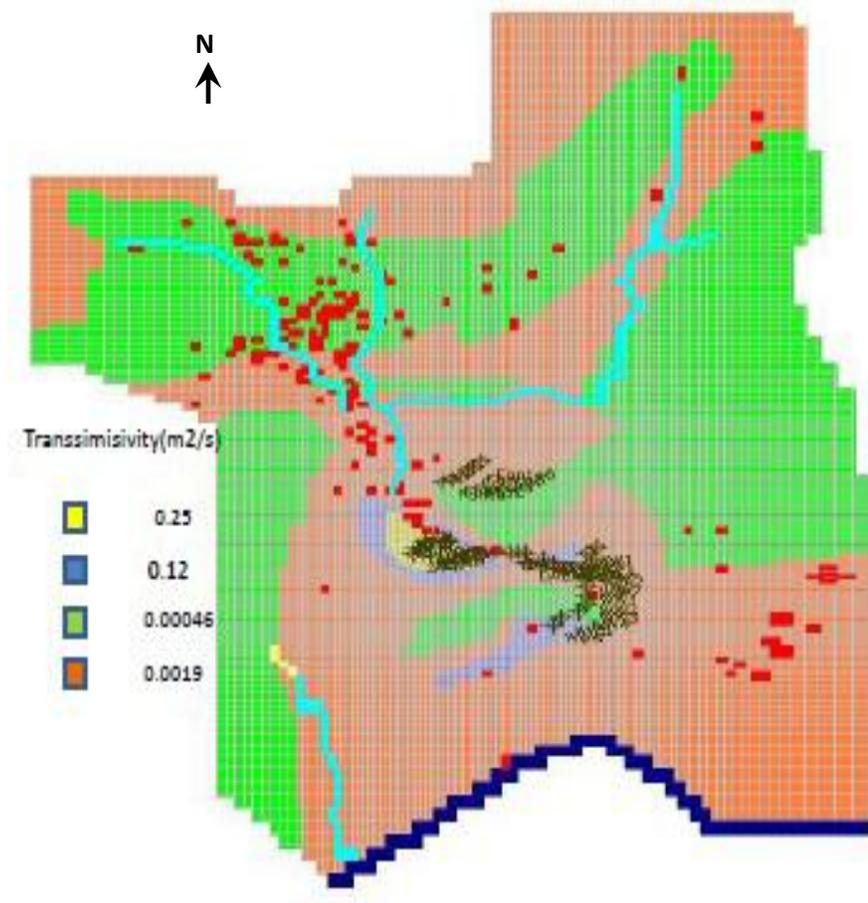


Figure 2-1 Grid structure and Transmissivity of regional model

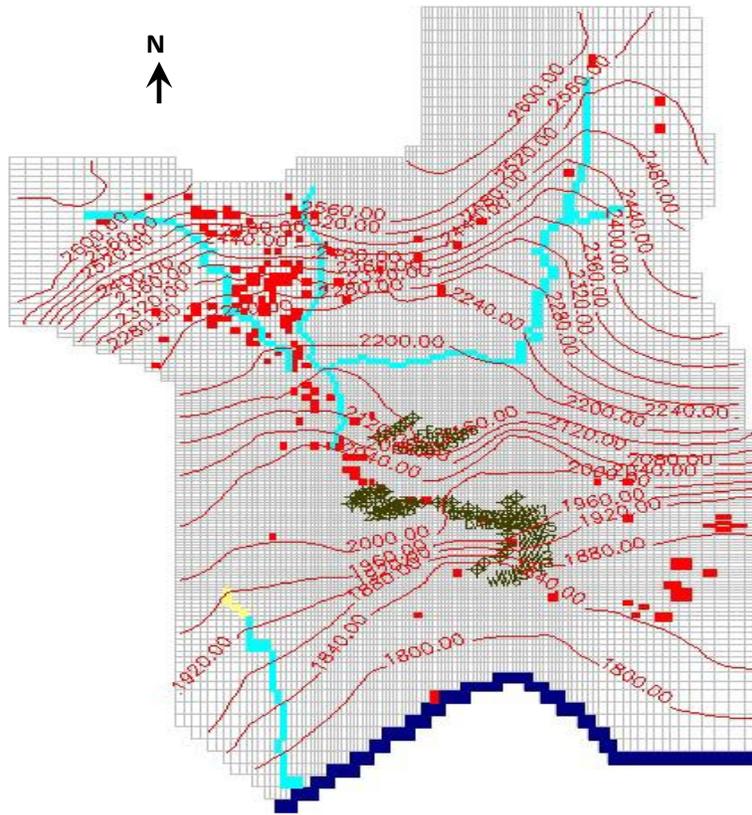


Figure 2-2 Hydraulic head distribution for regional model

The developed regional groundwater model provides the following water balance results (steady state).

Table 2-1 Groundwater balance of original model (m³/day)

Inflow to catchment			Outflow from catchment				
Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
281,059.2	518.4	281,577.6	187,228.8	24,451.2	5,270.4	64627.2	281,577.6
99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%

The water balance (table 2-1) after calibration of the steady state regional groundwater model shows that nearly all inflow to the model comes from natural recharge. Hence, 66.5% of the recharge is discharged by constant head boundary, 8.7% by wells, 1.9% by drains (springs) and 23% by river.

3 RESEARCH METHODOLOGY

3.1 Data Collection

The metrological (rainfall, temperature, sunshine) and river flow (discharge of Akaki river) data are collected from the metrological agency and Ministry of Water and Energy Resources. The location of reservoir sites which collects pumped water from the wells, cost for drilling of wells and operational costs are gathered from AAWSSA. The existing water abstraction rate from the wells and future plan of abstraction rate is also collected. In addition to this the regional groundwater model of Akaki catchment (presented at the end of previous section) is obtained from Addis Ababa water supply and sewerage Authority. Site inspection is also carried out to investigate status of the existing wells within the well fields.

3.2 Model development and adaptation

3.2.1 Introduction

Akaki catchment groundwater management model set up includes four well fields (Fanta, Dalota, Dukem up and Dukem down) in addition to Akaki well field. According to (BCEOM, 2000 and 2002) these are the well field for which potential well sites need to be investigated. These are eleven wells at Akaki well field for first phase, six wells at Fanta, six wells at Dalota, eight wells at Dukem up and six wells at Dukem down well fields (see Figure 3-1 below). In order to determine optimal abstraction rates and well spacing of these wells, drawdown constraints of 15 m, 20 m, 25 m and 30 m are used.

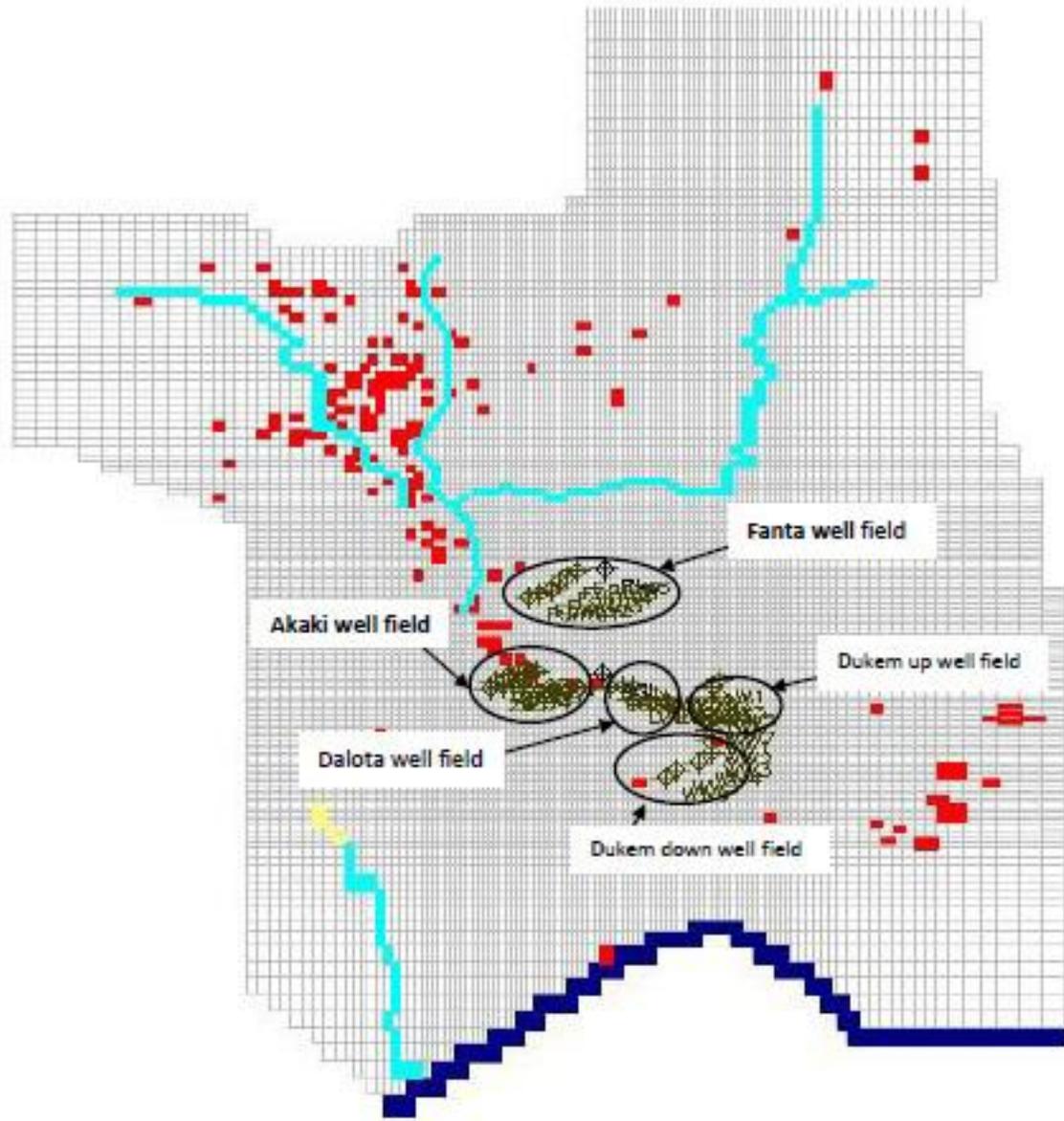


Figure 3-1 Location of potential well field areas

3.2.2 Adaptation of regional groundwater MODFLOW model

The regional ground water model of Akaki catchment is obtained from Addis Ababa Water Supply and Sewerage Authority (AAWSSA). The grid structure of the regional groundwater model is refined in order to have better information for all well fields. Grid spacing of 250m is maintained in the central area covering the well fields, but for the rest of the model area gradual increase of spacing of 350 m, 500 m, 750 m and 1000 m is used. The obtained water balance of refined regional groundwater model after simulation remains same as for the original model. In addition, the drawdown and hydraulic heads are also checked with the original model, and it is found that they are almost same as original model setup. The springs within the model area are

also kept. Particularly, Fanta spring is used for water supply sources for nearby areas. The flow of Fanta spring is estimated to be 20 to 30 l/s(BCEOM, 2000 and 2002) .The refined regional groundwater model water balance shows that: the inputs to the aquifer are natural recharge of 281059 m³/day and river leakage of 518.4 m³/day whereas the outputs of the aquifer are constant head outflow of 187229 m³/day (66.5%), wells 24451 m³/day (8.7%), drains (Fanta and Aba Samuel gorge springs) 5270 m³/day (1.9%) and river leakage of 64627 m³/day (23%). These are same values as presented in Table 2-1.

3.2.3 Development of groundwater management model using MODMAN and LINDO

As described in section 2.2 MODMAN (MODflow MANagemnet) is a FORTRAN code developed by HSI Geo Trans that adds optimization capability to the U.S.G.S. finite difference model for groundwater flow simulation in three dimensions, called MODFLOW-96. MODMAN enables to determine optimal location of wells and the abstraction or injection rates of wells, given a number of constraints. The groundwater management problem is formulated by creating appropriate input files, via a MODMAN pre-processor (Greenwald, 1998a), in which the desired objective function and constraints can be specified. The same pre-processor converts this specification into a file formatted according to the MPS format (Mathematical Programming System) that is used by LINDO in solving the optimization problem. In this way MODMAN will transform the groundwater management problem in to a linear or mixed-integer problem by using the response matrix technique.

The response matrix is generated based on the linear response theory (linear superposition) in groundwater systems, as described in section 2.2. In order to generate the unit response MODMAN calls MODFLOW once for each potential well location and these responses are included in the MPS input file, together with other required data for the problem specification.

Two different objective functions are formulated: maximization of abstraction rate and minimization of total cost (installation and operational costs for each well in the well fields). Detailed description of the actual objective function formulations will be given in chapter 4- Results and Discussion.

The specification of these objective functions is also done via the pre-processor and subsequently included in the MPS input file, although some modifications are required to be made directly in the MPS file(after its automatic generation by MODMAN) for the cost minimization objective. In this case the MPS input file is modified for inclusion of the coefficients of investment cost.

Finally, the constraints for the problem are also included in the MPS file via MODMAN pre-processor: drawdown constraints at different control locations, constraints on pumping rates the potential wells and balance constraints (in case of cost minimization).

Three different cases of specifying drawdown constraints at different control location are progressively tested, which give different solutions in terms of abstraction rates and well spacing for the selected wells. These cases are: 1) Imposing drawdown constraint of 15 m, 20 m, 25 m, and 30 m at location of each managed wells 2) Imposing drawdown constraint of 15 m, 20 m, 25 m, and 30 m at centre location of each well fields 3) Depending on the transmissivity value of the well location, for lower transmissivity zones drawdown constraint of 15 m, 20 m, 25 m, and 30 m are imposed at each managed well field whereas for those wells that are located in high transmissivity zones same drawdown constraints are imposed at selected centre location of the wells or well fields. The limiting drawdown values are selected from point of view of groundwater levels in the area, operational cost due to high drawdown and interest of AAWSSA.

After final formulation of the optimization problem and generation of the MPS file (step named as *mode 1* of MODMAN), the LINDO solver is called for obtaining the optimal solution. This optimal solution can then be converted to appropriate MODMAN format, through which MODMAN can generate MODFLOW input files with the optimal solution found (*mode 2* of MODMAN). The whole procedure for MODMAN is presented in Figure 3-2 below.

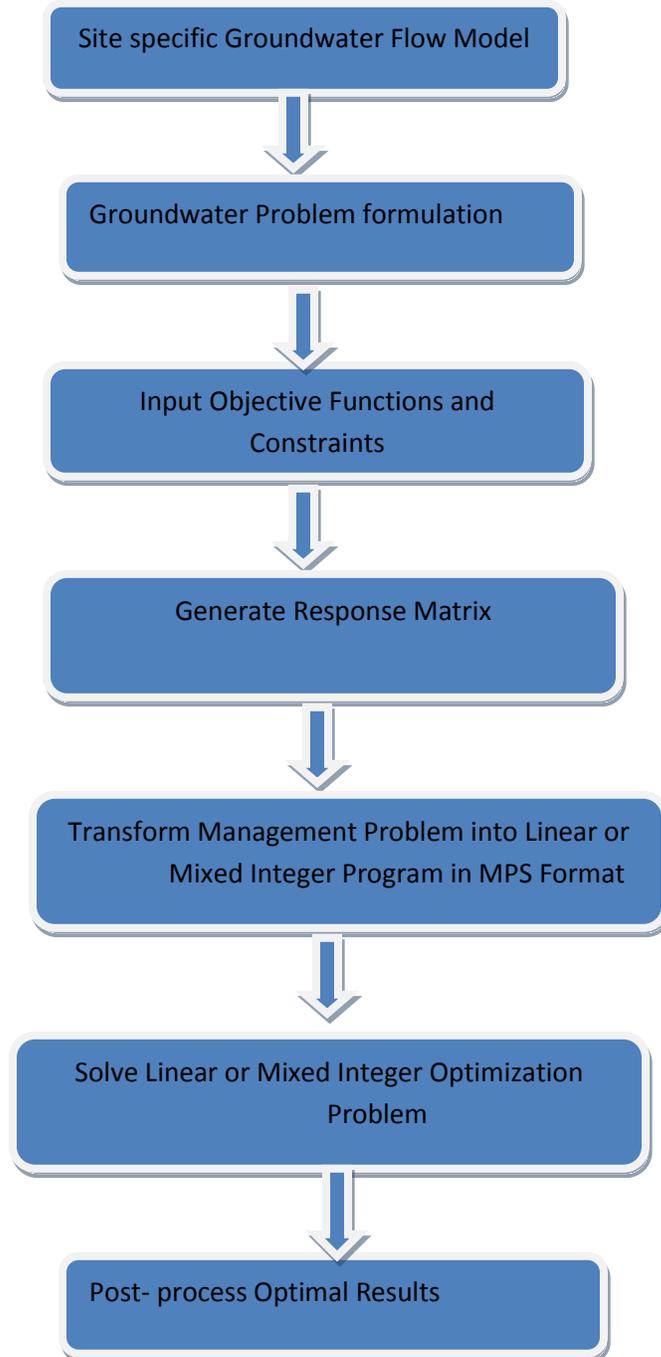


Figure 3-2 General flowchart of Optimization process (Greenwald, 1998)

4 RESULTS AND DISCUSSION

4.1 Steady State Condition

4.1.1 Problem formulation

The main problem in the study area is continuous decline of groundwater table due to over exploitation of groundwater in the area. This has caused increase in operational cost of wells in the well fields. To overcome this problem, finding optimal abstraction rate for each well is considered as one solution in this study. As explained in the previous section this groundwater optimization problem in the area is formulated via two objective functions and a number of constraints. These are: maximization of ground water abstraction rate and minimization of total cost of the well system in the area. Hydrologic constraints (drawdown and balance constraints) are imposed at different control locations. The full mathematical formulation of these optimization problems using the two objectives and the associated constraints is given below.

Objective one: Maximization of groundwater abstraction at Akaki, Fanta, Dalota, Dukem up and Dukem down well fields.

$$MAX: Z = \sum_{j=1}^N C_j \cdot Q_j \quad 4-1$$

Subject to drawdown constraints of:

$$DM = U_i - H_i = \sum_{j=1}^N R_{ij} \cdot Q_j \quad 4-2$$

$$DM_l \leq DM \leq DM_u \quad 4-3$$

Where:

C_j is cost coefficient, =1

DM_i is managed drawdown

U_i is unmanaged head

H_i is managed head

DM_l is lower drawdown limit at each control location, =0.0001

DM_u is upper drawdown limit at each control location=15 m, 20 m, 25 m, 30 m

Q_j is rate at managed well location j (negative for pumping)

$i = 1, 2, 3, \dots, N$ is control point location

$j = 1, 2, 3, \dots, N$ is pumping wells

Three different cases for drawdown constraint control location are considered. After analyzing the results of each case the best case is selected for cost minimization and unsteady state optimization. For sake of clarity, the three different cases of drawdown specification are repeated below.

- Drawdown constraint control location at each managed well location of each well field(see Figure 3-1)
- Drawdown constraint control location at centre of each well fields (in brackets- row and column coordinate of the chosen central location from the grid of the regional groundwater model)
 - Akaki well field at (99,54)
 - Fanta well field at(76,61)
 - Dalota well field at (102,78)
 - Dukem up well field at (104,92)
 - Dukem down well field at (117,87)
 - At Fanta spring (79,57)

The last constraint (at Fanta spring) is separate single cell constraint present in all cases which ensures that the drawdown at the spring location will be limited in such a way that the spring does not dry out.

- Drawdown constraint control location depending on the transmissivity of the aquifer of the area. For wells or well field areas in low transmissivity zones drawdown constraint is applied at each managed well, whereas for those wells or well field areas located in high transmissivity zones drawdown constraint is applied at centre of wells or well fields.

In addition to the above constraints depending on the existing abstraction rate of wells in the area minimum and maximum abstraction rates for each well is applied as described below.

$$0 \leq Q_j \leq Q_jmax \quad 4-4$$

Where:

Q_j is rate at managed well location j (negative for pumping)

Q_jmax is the maximum abstraction rate for each of managed wells = 4320 m³/day

Objective two: Minimization of cost of well system to obtain optimal water abstraction rate and drawdown at certain level. In case of this objective investment cost of wells (drilling and cost for connection of each well to reservoir location) and operational cost of each well is considered.

$$min: Z = \sum_{j=1}^N (C_j \cdot Q_j + d_j \cdot I_j) \quad 4-5$$

Where:

C_j is cost per unit pumping rate at well j (negative value for pumping well)

d_j is additional cost (well and pipe installation cost) at well j

Q_j is rate at well j (negative for pumping)

I_j is 1 if well is active, if not zero

The coefficient C_j is calculated from pumping rate cost whereas d_j is calculated from drilling and installation cost of wells, construction and installation of pipes. In order to determine the installation cost for pipes that connect wells to main reservoir; shortest distance (straight line) is selected between the location of each well and location of main reservoir. The coefficients for each of the costs are obtained as follows.

$$d_j = \sum_{j=1}^N (w_c + P_c) \quad 4-6$$

Where d_j total cost of drilling and pipe installation in ETB

w_c drilling cost of each well whereas P_c is pipe cost in ETB

$$C_j = \sum_{j=1}^N Pl * \text{cost per unit meter length} \quad 4-7$$

Where c_j total operational cost in ETB

pl pipe length from well location to reservoir whereas cost per unit meter of length is

266.4 ETB

In order to determine the value for unit pumping cost of wells C_j the following assumptions are made: 1) the average pumping rate of each well is 30 l/s; 2) pumping head is 60m; 3) life time of wells is 30year and 8hour working time is used for each well. Therefore, the power cost required to unit rate pumping (operational cost) is 262,800ETB for 30years (See the analysis of drilling and installation costs in Appendix 3).

The drawdown constraint locations, the minimum and maximum abstraction rate of each well are same as those for the maximization of abstraction rate objective. One conditional difference is that a balance constraint is used to limit the total abstraction rate of wells which is expressed as follows:

$$\sum_{j=1}^N Q_j \leq Q \quad 4-8$$

Where:

Q_j is rate at well j

Q is the total abstraction rate to be specified in m^3/day

The maximum balance constraint Q applied for 15 m, 20 m, 25 m and 30 m is 20,736 m^3/day , 23,328 m^3/day , 26,784 m^3/day and 29,376 m^3/day respectively for total cost minimization objective. These values are same as the results of total abstraction rates of first objective for their respective drawdown constraints (maximization of abstraction rates).

Finally, integer constraint is also applied to select X out of Y wells. In this case $X \leq 36$ which is the total number of managed wells. All objective function and constraint specification is done via MODMAN pre-processor, except for drilling and pipe installation cost of each managed wells, which are added by modifying MPS file manually after execution of MODMAN mode 1.

4.1.2 Results of maximization of abstraction rate

For management of groundwater problem at Akaki catchment well fields, three cases of drawdown constraint control location are used to determine the best optimal abstraction rate and well spacing in the area. According to these cases the result for each case is shown below.

Case One: Drawdown constraints are imposed at each managed well locations. The drawdown constraint of 15 m, 20 m, 25 m, and 30 m are imposed to obtain optimal abstraction rate and well spacing of each well field. Table 4-1 shows result of optimal abstraction rate and wells that are selected. The drawbacks of this case are the fact that large number of potential wells are selected(except few wells close to Fanta spring constraint) and that many of these are with optimal abstraction rates which are small compared to the pre-determined abstraction rate capacities of the wells in the area. These drawbacks are confirmed for all drawdown constraints, except for the last one of 30 m. In this case, out of all wells in Akaki well field only one is selected in the optimal solution (see Table 4-1). For the other well fields the situation is the same as for the lower drawdown constraints.

Figure 4-1 presents the summary of potential well locations, drawdown location, and selected wells for drawdown constraints 15 m. (For clarity, large symbols are used, even though these locations are per one modelling cell). The figure presents an enlarged view from the model that covers only the area where the well fields are located (The red cells contain pre-existing well locations which are non-managed wells).

Table 4-1 Selected wells and optimal abstraction rate in steady state_case1

Selected wells	Optimal abstraction rates (m ³ /day)			
	Drawdown imposed at each centre location (m)			
	15 m	20 m	25 m	30 m
Akaki_276	-321.4	-470	-617.8	--
Akaki_277	-228.1	-326.6	-426	--
Akaki_278	-278.2	-394	-509.8	--
Akaki_279	-578.9	-852.8	-1126.7	--
Akaki_284	-231.6	-323.1	-414	--
Akaki_285	-793.2	-1152.6	-1511.1	--
Akaki_286	-1180.2	-1696	-2211	--
Akaki-287	-385.3	-535.7	-685.2	-1400.5
Akaki-290	-448.4	-639.4	-831.2	--
Akaki-291	-1236.4	-1745.3	-2254.2	--
Fanta_6	-3952	-2097.8	-243.7	--
Dalota_1	-37.2	-55.3	-73.4	-2716.4
Dalota_2	-21.6	-32.8	-44.1	-712
Dalota_3	-31.1	-47.5	-64.8	-323.1
Dalota_4	-169.3	-244.5	-319.7	-4320
Dalota_5	-251.4	-370.7	-490	-1563
Dalota_6	-383.6	-560.7	-738	-1434.2
Dukem up_1	-613.4	-872.6	-1132.7	-1394.5
Dukem up_2	-196.1	-281.7	-367.2	-473.5
Dukem up_3	-287.7	-402.6	-517.5	-638.5
Dukem up_4	-103.7	-147.7	-5192.7	-262.7
Dukem up_5	-731	-1013.5	-1296	-1587.2
Dukem up_6	-262.7	-364.6	-466.6	-576.3
Dukem up_7	-1091.2	-1511.1	-1932	-2370.8
Dukem up_8	-228.1	-318.8	-409.5	-520.1
Dukem down_1	-553	-771.6	-991	-1230.3
Dukem down-2	-133.1	-186.6	-240.2	-299.8
Dukem down_3	-203.9	-287.7	-371.5	-462.2
Dukem down_4	-351.7	-504.6	-658.4	-827
Dukem down_5	-1098.1	-1587.2	-2075.3	-2612.7
Dukem down_6	-4320.0	-4320.0	-4320.0	-4320.0
Total abstraction rate (m³/day)	-20,701.4	-24,115.1	-27,530.5	-30,045
Total cost (Million ETB)	54	54	54	37

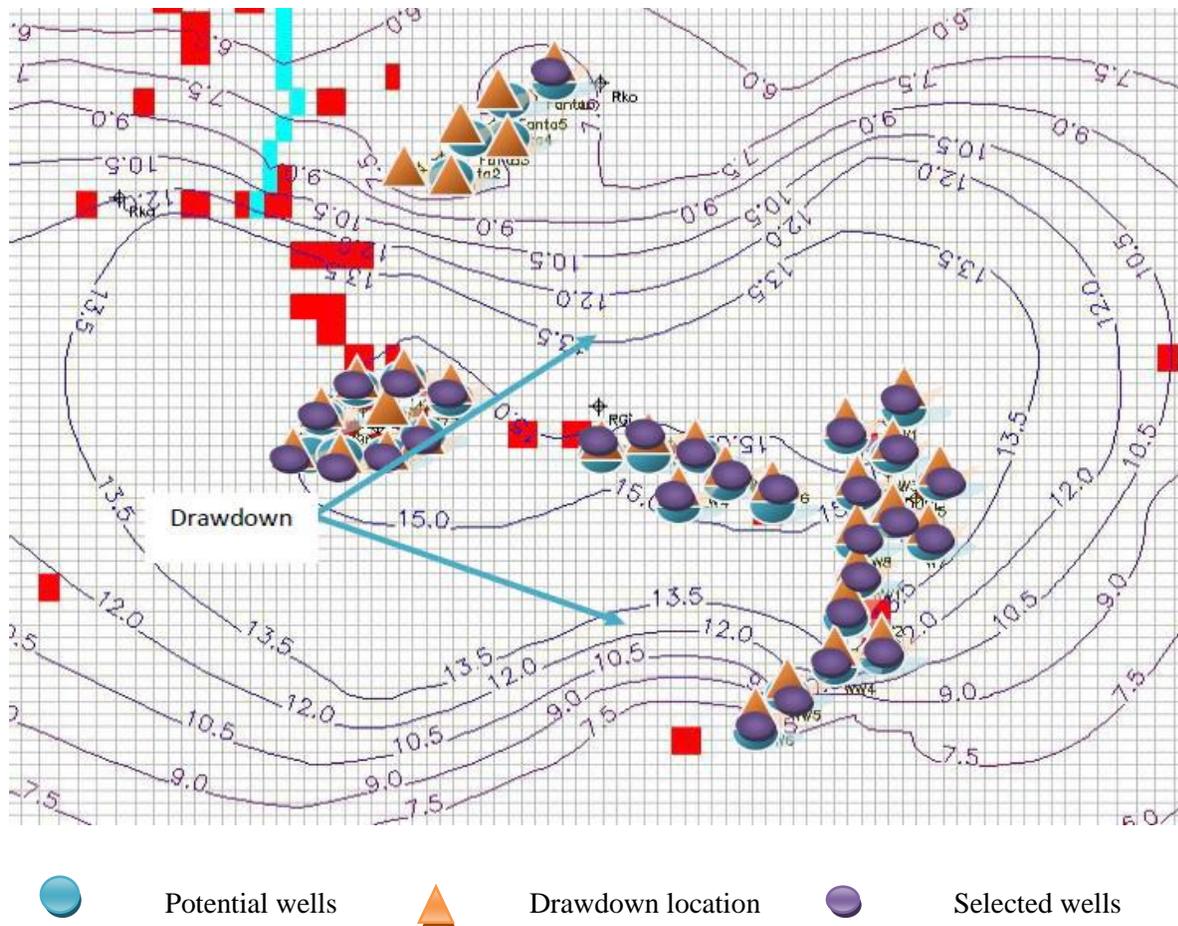


Figure 4-1 Drawdown and selected wells for 15 m drawdown constraint_case1

Table 4-2 Groundwater balance for 15 m drawdown constraint of case1 (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059	518	281,578	187,228.8	24,451	5,270	64627	281,577.6
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 15 m drawdown	281,059	578.0	281,578	175,755	45,161	3050	57,698	281,577.6
	99.8%	0.2%	100%	62.4%	16%	1.1%	20.4%	100%
Difference (m ³ /day)	0	59.6	0	-11,474	20,710	-2221	-6929	0

The water balance (table4-2) shows that 62.4% of recharge is discharged by constant head boundary, 16% by wells, 1.1% by drains and 20.4% by river flow. When compared with the water balance from the original model(Table 2-1) the result shows that increase of abstraction rate of wells by 20,710 m³/day has caused decrease of outflow to river by 6929 m³/day , outflow to drains (springs) by 2221 m³/day, and outflow to constant head boundary by 11,474 m³/day.

In Figure 4-2 to 4-4, and tables 4-3 to 4-5, corresponding results are presented for drawdown constraints of 20 m, 25 m, and 30 m.

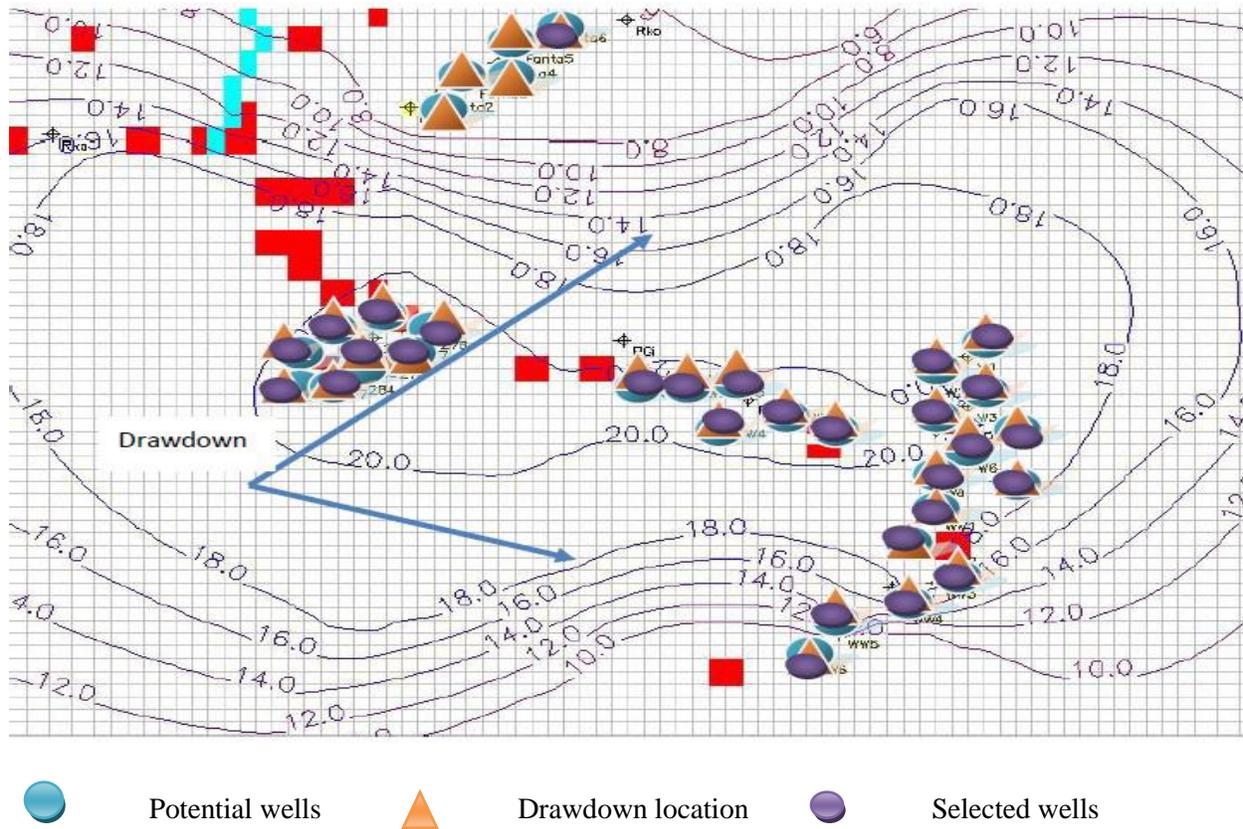


Figure 4-2 Drawdown and selected wells for 20 m drawdown constraint_case1

The water balance (table4-3) shows that: 61.4% recharge is discharged by constant head boundary, 17.2% by wells, 1.0% drains and 20.4 % by the river. When compared with the original steady state model the result shows that increase of abstraction rate by 24,123 m³/day has caused decrease of outflow to river by 7258 m³/day, outflow to drains(springs) by 2437 m³/day and outflow to constant head boundary by 14,360 m³/day.

Table 4-3 Groundwater balance for 20 m drawdown constraint of case1 (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,228.8	24,451	5,270	64627	281,578
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 20 m drawdown	281,059.2	570.2	281,629	172,869	48,574	2834	57,370	281,647
	99.8%	0.2%	100%	61.4%	17.2%	1.0%	20.4%	100%
Difference (m ³ /day)	0	51.8	0	-14,360	24,123	-2437	-7258	69

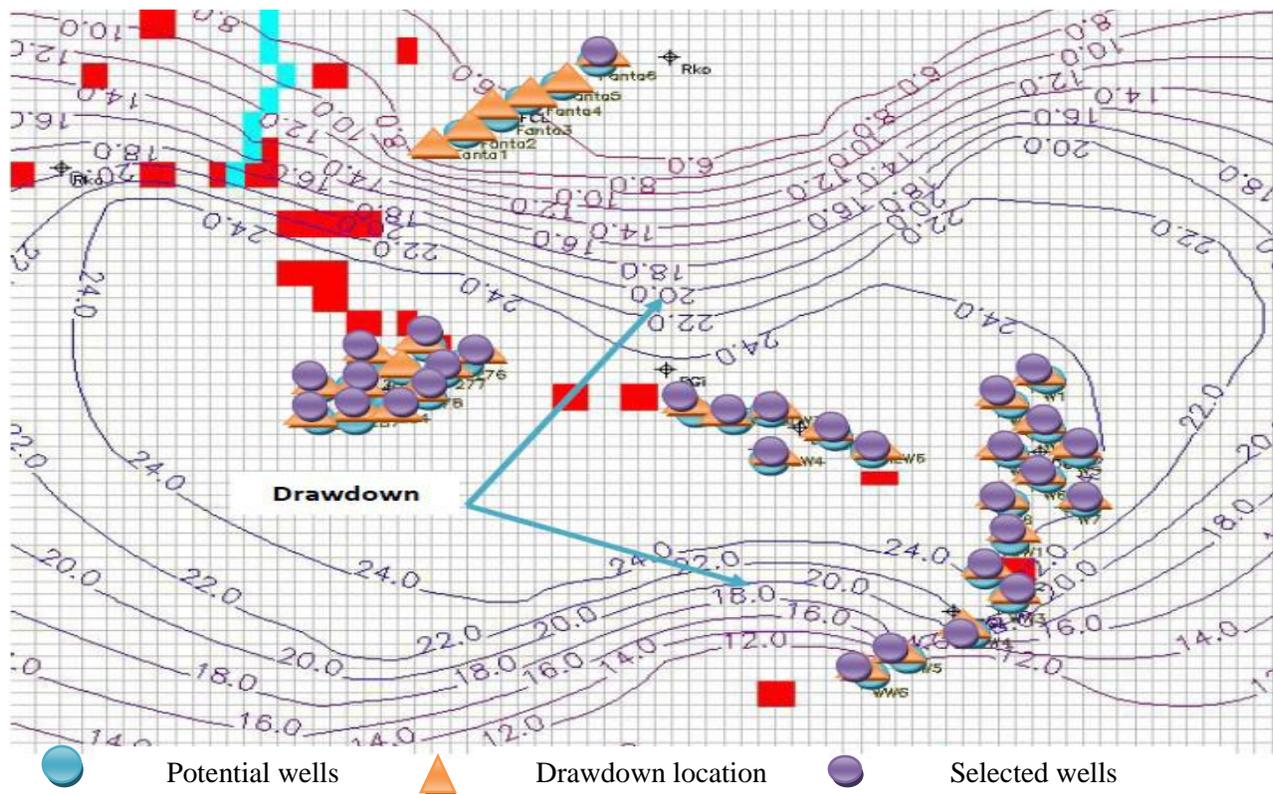


Figure 4-3 Drawdown and selected wells for 25 m drawdown constraint_case1

The water balance (table4-4) shows that: 60.4% of recharge is discharged by constant head boundary, 18.5% by wells, 0.9% by drains and 20.2% by the river flow. When compared with the original steady state model the result also shows that increase of abstraction rate by 27, 536 m³/day has caused decrease of outflow to river by 7545 m³/day, outflow to constant head boundary by 17,254 m³/day and outflow to drains(springs) by 2658 m³/day.

Table 4-4 Groundwater balance for 25 m drawdown constraint of case1 (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,228.8	24,451	5,270	64627	281,578
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 25 m drawdown	281,059.2	570.2	281,629	169,974.7	51,987	2613	57,082	281,656
	99.8%	0.2%	100%	60.4%	18.5%	0.9%	20.2%	100%
Difference (m ³ /day)	0	51.8	51	-17,254	27,536	-2658	-7545	78

The water balance (table4-5) shows that: 59.5% of recharge is discharged by constant head boundary, 0.9% by drains and 20.2% by river flow. When compared with original steady state model the result also shows that increase abstraction rate by 30,050 m³/day has caused decrease of outflow to river by 7733 m³/day , outflow to drains(springs) by 2678 m³/day and outflow to constant head boundary by19,613 m³/day.

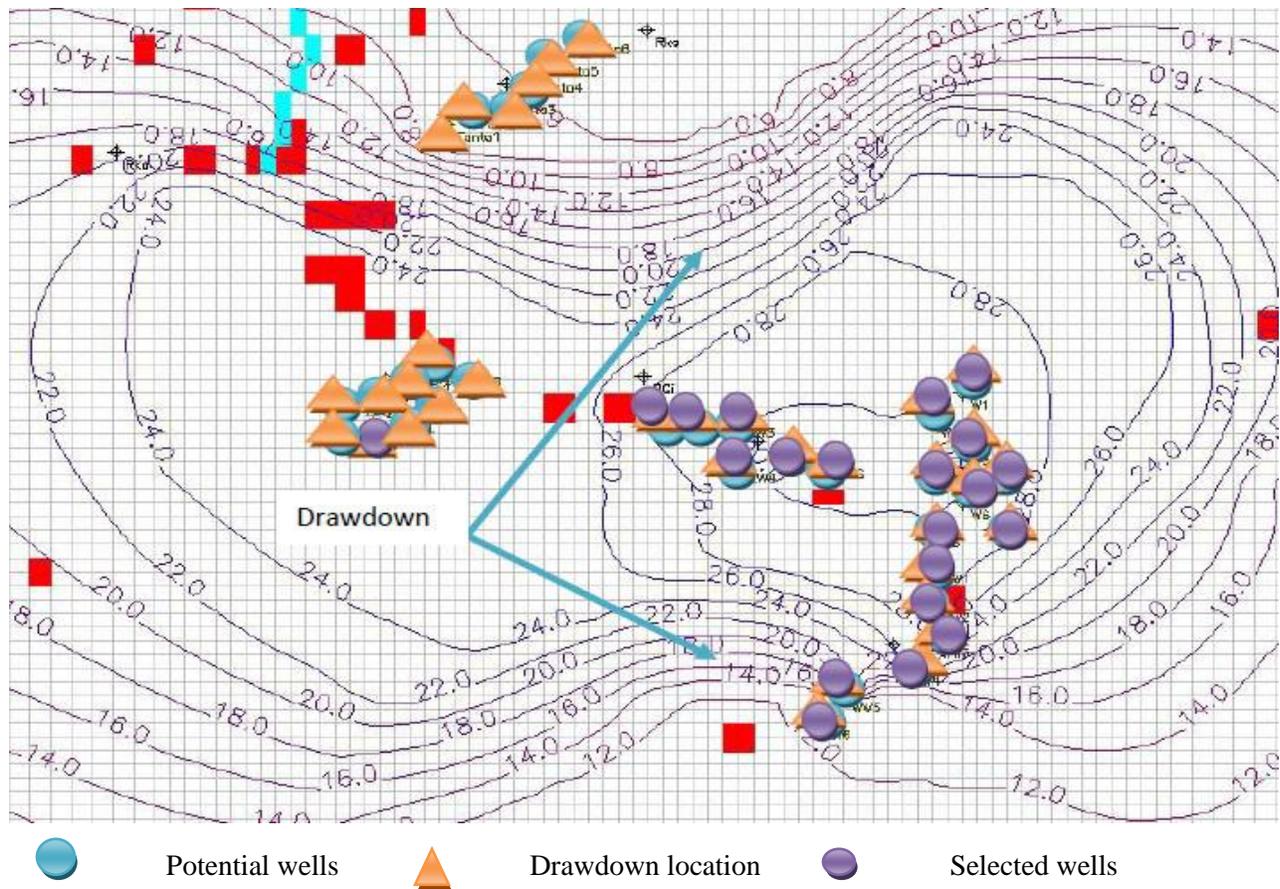


Figure 4-4 Drawdown and selected wells for 30 m drawdown constraint_case1

Table 4-5 Groundwater balance for 30 m drawdown constraint of case1 (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,228.8	24,451	5,270	64627	281,577.6
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 30 m drawdown	281,059.2	570.2	281,629	167,616.0	54,501	2592	56,894	281,603.5
	99.8%	0.2%	100%	59.5%	19.4%	0.9%	20.2%	100%
Difference (m ³ /day)	0	51.8	51	-19,613	30,050	-2678	-7733	26

Case two: Drawdown constraint control locations are imposed at centre location of each well field.

Because of the identified problems in case one of drawdown specification as described in previous section, a second case was chosen for specifying the drawdown constraints: in this case a centre location point is selected for each of well fields as drawdown constraint control location. Table 4-6 shows selected wells and optimal abstraction rate of each selected wells.

Table 4-6 Selected wells and optimal abstraction rate for steady state case-2

Selected wells	Optimal abstraction rates(m ³ /day)			
	Drawdown imposed at each centre location (m)			
	15m	20m	25m	30m
Akaki_287	--	--	-1874.0	-496
Akaki_290	-1175	-3647	-4320	--
Akaki_291	-4320	-4320	-4320	-4320
Fanta_6	-3944.2	-2089.2	-236.8	--
Dukem up_1	--	--	--	-2537.6
Dukem up_7	-2150.5	-2755.3	-3887.1	-4320
Dukem up_8	--	--	--	-4320
Dukem down_1	-2548.8	-3774	-4320	-4320
Dukem down_3	-330	-1258	-2252.5	-3386
Dukem down_5	-4320	-4320	-4320	-4320
Dukem down_6	-4320	-4320	-4320	-4320
Total abstraction rate (m ³ /day)	-23,108.5	-26,483.3	-29,850.3	-32,339.5
Total cost (Million ETB)	17	17	19	19

Figures 4-5 to 4-8 present the optimal solutions for this case for drawdown constraints of 15, 20, 25 and 30 m respectively, with the same symbols as for the first case of drawdown specification. Tables 4-7 to 4-11, present the water balance results from the optimal solutions for the same drawdown constraints.

The water balance (table4-7) shows that: 61.5% of recharge is discharged by constant head boundary, 16.9% by wells, 1.1% by drains and 20.2% by river. When compared with the original steady state model the result shows that the increase of abstraction rate by 23,115 m³/day has caused decrease of outflow to river by 6978 m³/day , outflow to drains(springs) by 2246 m³/day and outflow to constant head boundary by 13,807 m³/day.

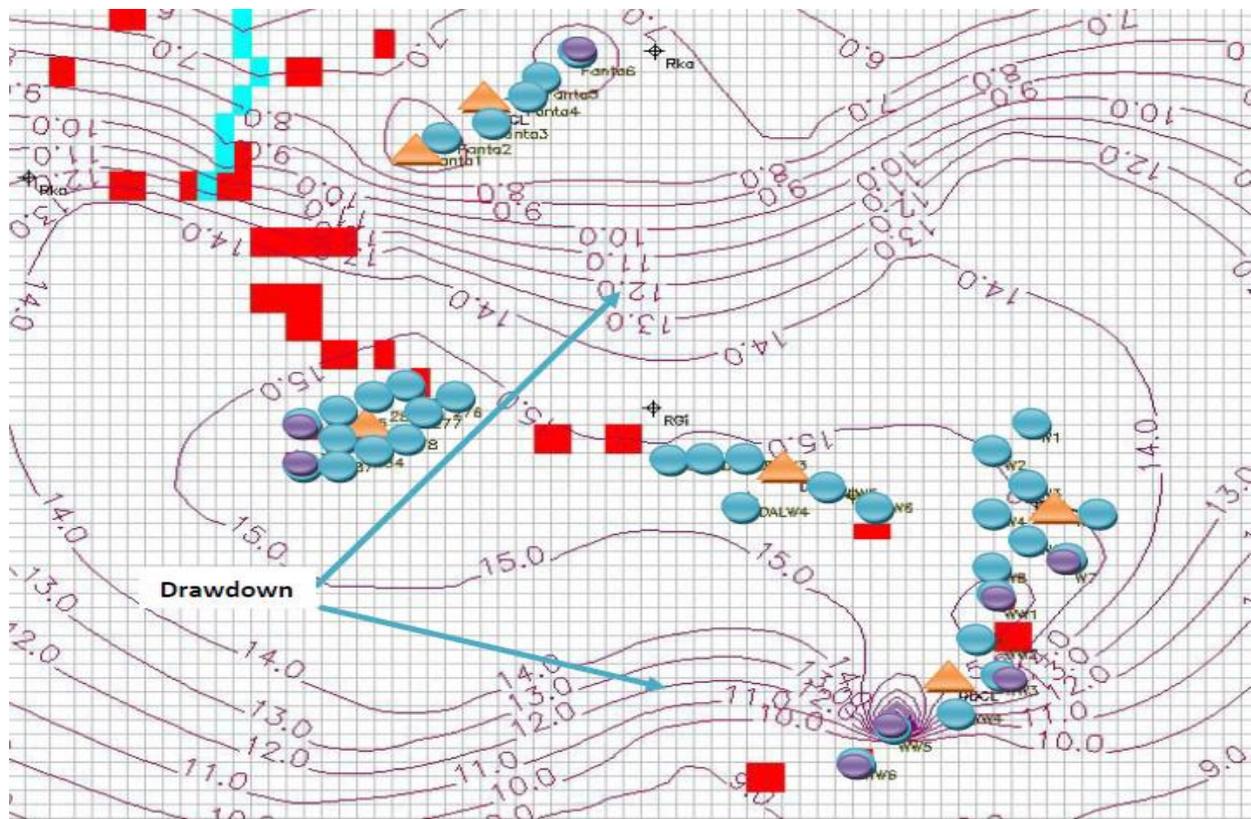


Figure 4-5 Drawdown and selected wells for 15 m drawdown constraint_case2

Table 4-7 Groundwater balance for 15m drawdown constraint of case2 (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,228.8	24,451	5,270	64,627	281,577.6
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 15 m drawdown	281,059.2	578.9	281,838	173,422.1	47,566	3,024	57,650	281,661.4
	99.8%	0.2%	100%	61.5%	16.9%	1.1%	20.2%	100%
Difference (m ³ /day)	0	60.5	260	-13,807	23,115	-2,246	-6,978	83.8

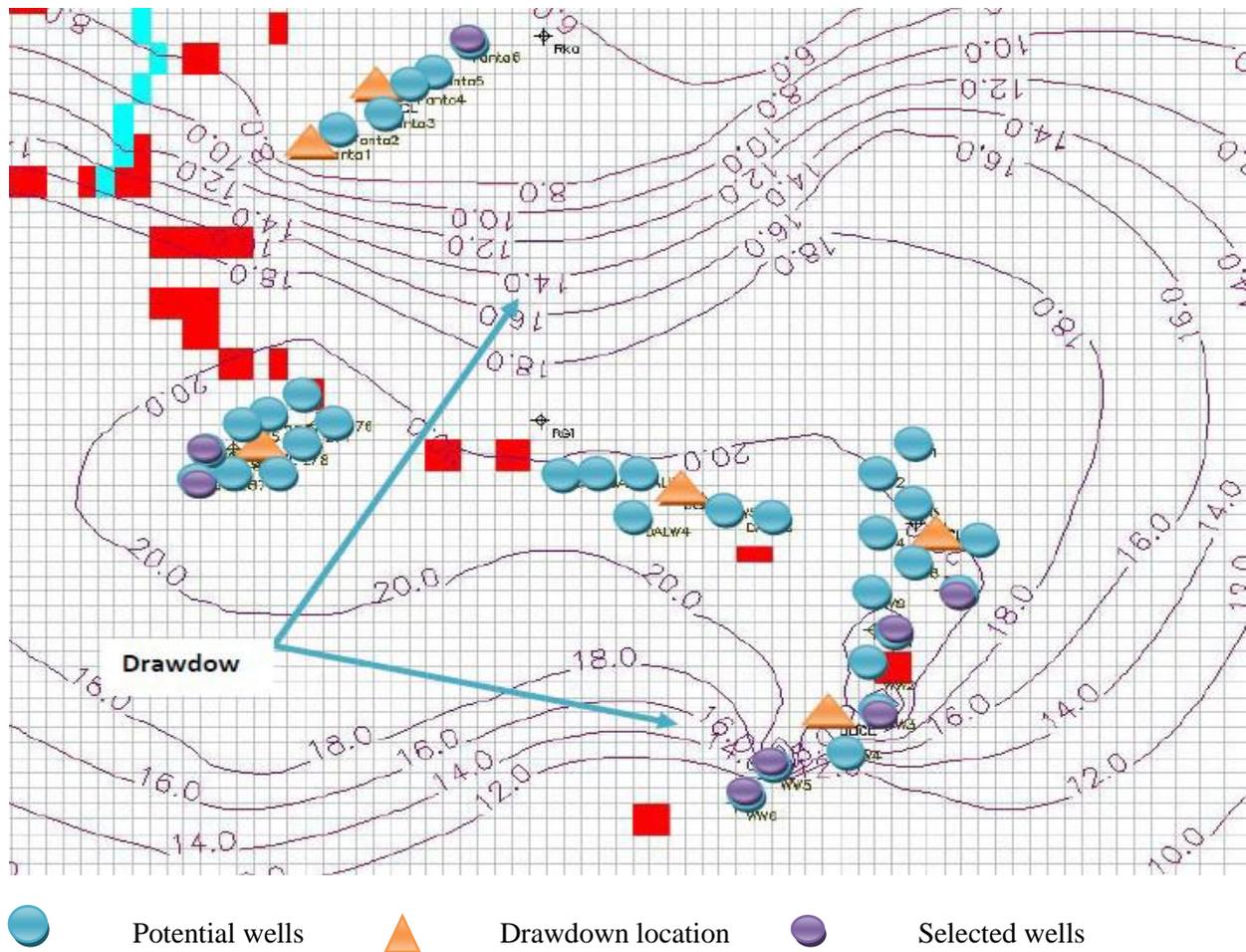


Figure 4-6 Drawdown and selected wells for 20 m drawdown constraint_case2

The water balance (table4-8) shows that: 60.6% of recharge is discharged by constant head boundary, 18.1% by wells, 0.9% by drains and 20.4% by river flow. When compared with the water balance of original steady state model the result shows that the increase of abstraction rate by 26,490 m³/day has caused decrease of outflow to river by 7301 m³/day, outflow to drains (springs) by 2462 m³/day and outflow to constant head boundary by 16,649 m³/day

Table 4-8 Groundwater balance for 20 m drawdown constraint of case2 (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,228.8	24,451	5,270	64627	281,577.6
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 20 m drawdown	281,059.2	570.24	281,629	170,579.5	50,941	2808	57,326	281,655.3
	99.8%	0.2%	100%	60.6%	18.1%	0.9%	20.4%	100%
Difference (m ³ /day)	0	51.8	51	-16,649	26,490	-2462	-7301	77.7

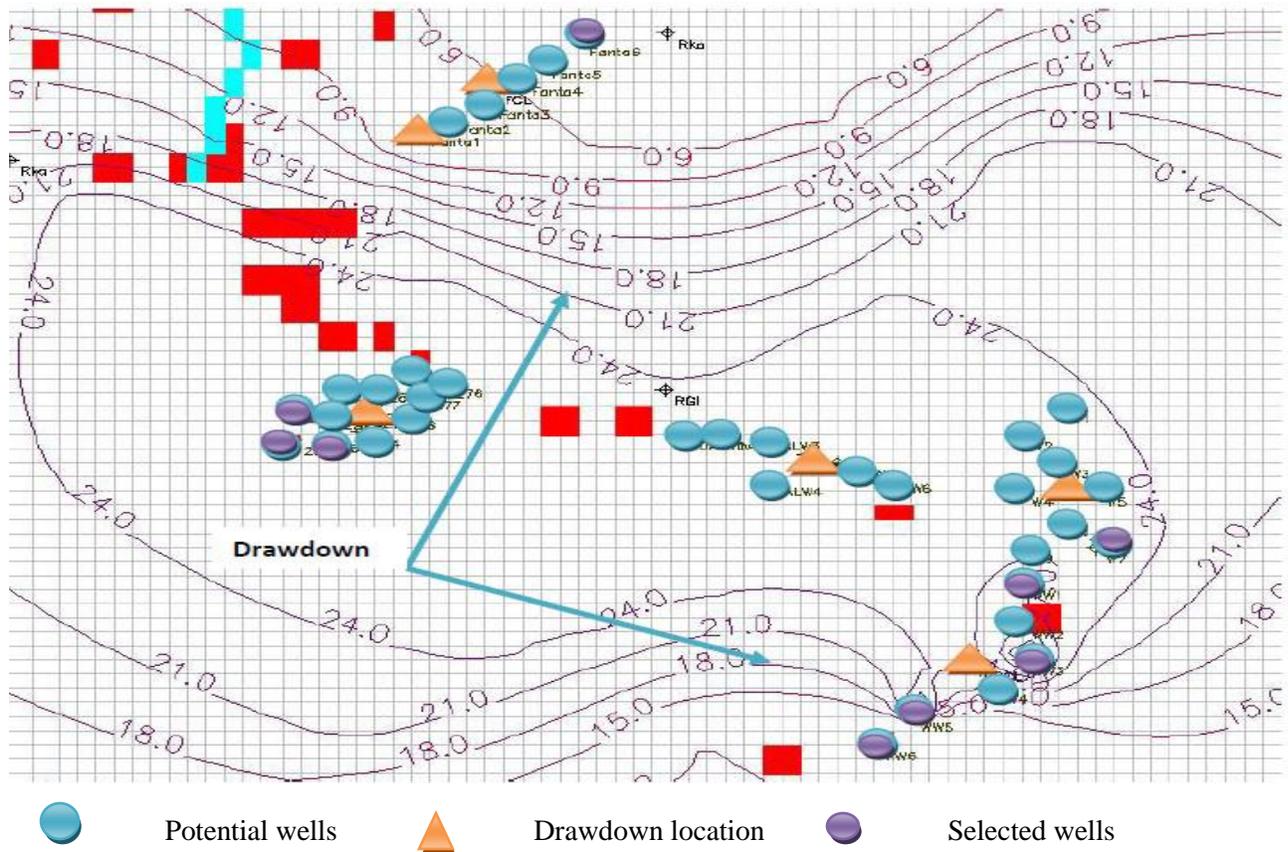


Figure 4-7 Drawdown and selected wells for 25 m drawdown constraint_case2

The water balance (table4-9) shows that: 59.6% of recharge is discharged by constant head boundary, 19.3% by wells, 0.9% by drains and 20.3% by river flow. When compared with water balance of the original steady state model, the result shows increase of abstraction rate by 29,860 m³/day has caused decrease of outflow to river by 7591 m³/day, outflow to drains(springs) by 2678 m³/day and outflow to constant head boundary by 19,501 m³/day.

Table 4-9 Groundwater balance for 25 m drawdown constraint of case2 (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,228.8	24,451	5,270	64627	281,577.6
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 25 m drawdown	281,059.2	570.24	281,629	167,728.3	54,311	2592	57,036	281,667.4
	99.8%	0.2%	100%	59.6%	19.3%	0.9%	20.3%	100%
Difference (m ³ /day)	0	51.8	51	-19,501	29,860	-2678	-7591	89.8

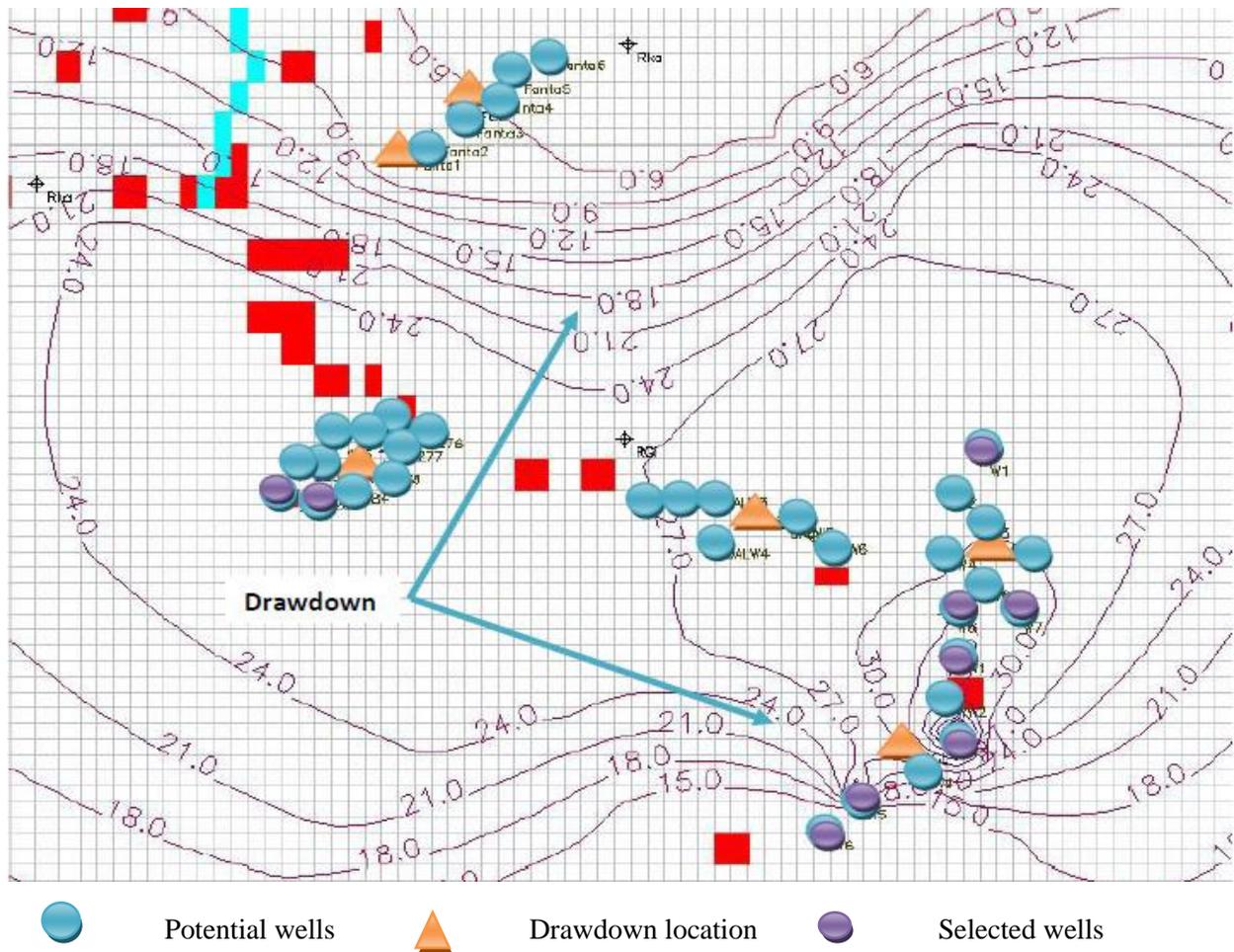


Figure 4-8 Drawdown and selected wells for 30 m drawdown constraint_case2

The water balance (table 4-10) shows that: 58.7% of recharge is discharged by constant head boundary, 20.2% by wells, 0.9% by drains and 20.2% by river flow. When compared with water balance of original steady state model, the result shows increase of abstraction rate by 32,348 m³/day has caused decrease of outflow to constant head boundary by 21,773 m³/day, outflow to drains(springs) by 2704 m³/day and outflow to river flow by 7776 m³/day.

Table 4-10 Groundwater balance for 30 m drawdown constraint of case2 (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,228.8	24,451	5,270	64627	281,577.6
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 30 m drawdown	281,059.2	570.24	281,629	165,456	56,799	2566	56,851	281,672.6
	99.8%	0.2%	100%	58.7%	20.2%	0.9%	20.2%	100%
Difference (m ³ /day)	0	51.8	51	-21,773	32,348	-2704	-7776	95

All these results indicate that with this case of drawdown specification may be better compared to case one. The overall maximum abstraction rate is higher for all four drawdown constraints. Moreover in all cases this higher maximum abstraction rate is achieved with smaller number of selected wells in the optimal solutions (obviously this would reduce the total cost of the solution). Only few wells in the optimal solutions are with small abstraction rates, whereas most of them are pumping with significant rates within the range expected from the pre-determined well capacities.

However, there is still one drawback of this approach: After simulating the obtained optimal abstraction rates for this case with the regional groundwater MODFLOW model of the area, the drawdown result is actually higher as compared to imposed drawdown constraints in some well locations. In other words, because the drawdown constraints are not imposed on every well location some well locations show draw downs higher than the one imposed in the centre of the well field. This is particularly the case for wells located in zones of lower aquifer transmissivity. Therefore, the obtained optimal abstraction rates of wells in the well fields for this case cannot be recommended, and a new drawdown constraint specification is required.

Case three: Drawdown constraint control locations are imposed at centre location of each well field and at managed well location depending on the transmissivity of wells and well fields.

Given the obtained results from cases 1 and 2, the third case for drawdown specification is in fact a kind of combination of the first two cases. Whether a constraint will be specified at the centre of a well field or at a well location now depends on the transmissivity value of the cell in which a

well is located. For high transmissivity zones drawdown constraint is imposed at the centre of the well fields. For wells in lower transmissivity zones, drawdown constraints are imposed at each well location. In comparison with the above cases, this case seems to perform better. The total maximum abstraction rates are comparable to case one (slightly higher), but the number of selected wells is smaller and most of them are with significant rates within the range expected from the pre-determined well capacities. At the same time there is no violation of drawdown constraints at any well location. This case therefore seems to be the best way of specifying drawdown constraints in the formulated optimization problems. Table4-11 shows abstraction rates and selected wells of this case.

Table 4-11 Optimal abstraction rate for case-3

Selected wells	Optimal abstraction rates(m ³ /day)			
	Drawdown imposed at each centre location (m)			
	15m	20m	25m	30m
Akaki_287	-1682.2	-4279.4	-4320.0	-2169.5
Akaki_290	--	--	-2553.1	--
Akaki_291	-4320.0	-4320.0	-4320.0	--
Fanta_6	-3951.9	-2101.3	-248.0	--
Dalota_1	--	--	--	-3303.9
Dalota_2	--	--	--	-155.5
Dalota_4	--	--	--	-4320.0
Dukem up_1	-748.2	-1070.5	-1393.6	-1595.8
Dukem up_2	--	--	--	-1263.2
Dukem up_3	-438.9	-623.8	-809.6	-918.4
Dukem up_5	-797.5	-1108.5	-1420.4	-1727.1
Dukem up_7	-994.5	-1373.8	-1753.0	-2128.0
Dukem down_1	-1822.2	-2578.2	-3337.6	-4320.0
Dukem down_2	-63.1	-87.3	-111.5	-131.3
Dukem down_3	-178.0	-251.4	-324.0	-400.0
Dukem down_4	-345.6	-496.8	-648.0	-813.9
Dukem down_5	-1093.0	-1580.3	-2067.6	-2602.4
Dukem down_6	-4320.0	-4320.0	-4320.0	-4320.0
Total abstraction rate (m ³ /day)	-20,755	-24,191	-27,626	-30,169
Total cost (Million ETB)	28	28	29	28

Figures 4-9 to 4-12 present the optimal solutions for this case for drawdown constraints of 15, 20, 25 and 30 m respectively, with the same symbols as for the first and second case of drawdown specification. Tables 4-12 to 4-15, present the water balance results from the optimal solutions for the same drawdown constraints.

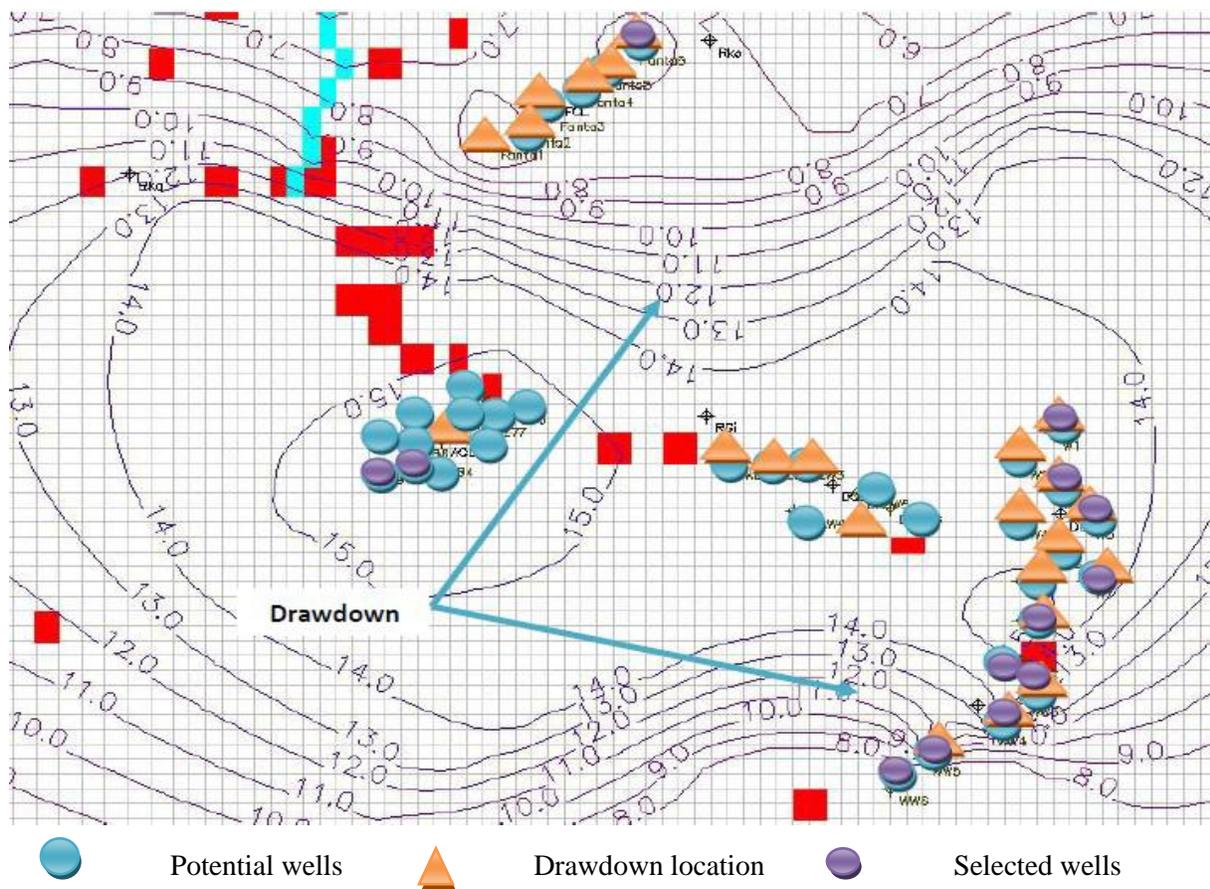


Figure 4-9 Drawdown and selected wells for 15 m drawdown constraint_case3

Table 4-12 Groundwater balance for 15 m drawdown constraint of case3 (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,228.8	24,451	5,270	64627	281,577.6
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 15 m drawdown	281,059.2	570.24	281,629	175,703	45,213	3049.9	58,562	282,528.0
	99.8%	0.2%	100%	62.4%	16.1%	1.1%	20.8%	100%
Difference (m ³ /day)	0	51.8	51	-11,526	20,762	-2221	-6065	950.4

The water balance (table4-12) shows that: 62.4% of recharge is discharged by constant head boundary, 16.1% by wells, 1.1% by drains and 20.8% by river flow. When compared with water balance of original steady state model ,the result shows increase of abstraction rate by 20,762 m³/day 84.9% has caused decrease of outflow to river by 6065 m³/day , outflow to drains (springs) by 2221 m³/day and outflow to constant head boundary by 11,526 m³/day.

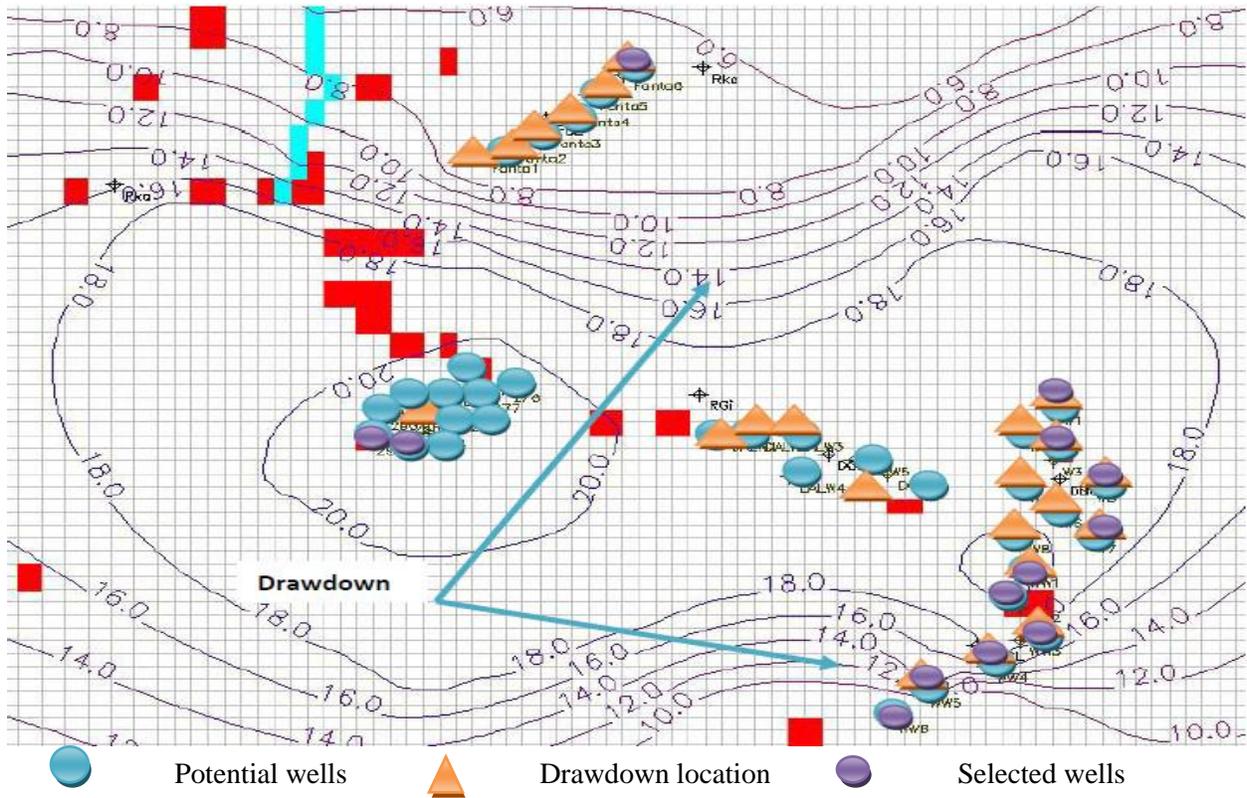


Figure 4-10 Drawdown and selected wells for 20 m drawdown constraint_case3

Table 4-13 Groundwater balance for 20 m drawdown constraint of case3 (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,228.8	24,451	5,270	64627	281,578
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 20 m drawdown	281,059.2	570.2	281,629	172,800	48,652	2834	57,378	281,664
	99.8%	0.2%	100%	61.4%	17.3%	1.0%	20.4%	100%
Difference (m ³ /day)	0	51.8	51	-14,429	24,201	-2437	-7249	86

The water balance (table4-13) shows that: 61.4% of recharge discharged by constant head boundary, 17.3% by wells, 1% by drains and 20.4% by river. When compared with water balance of original steady state model, the result shows the increase of abstraction rate by 24,201 m³/day has caused decrease of outflow to river by 7249 m³/day, outflow to drains (springs) by 2437 m³/day and outflow to constant head boundary by 14,429 m³/day.

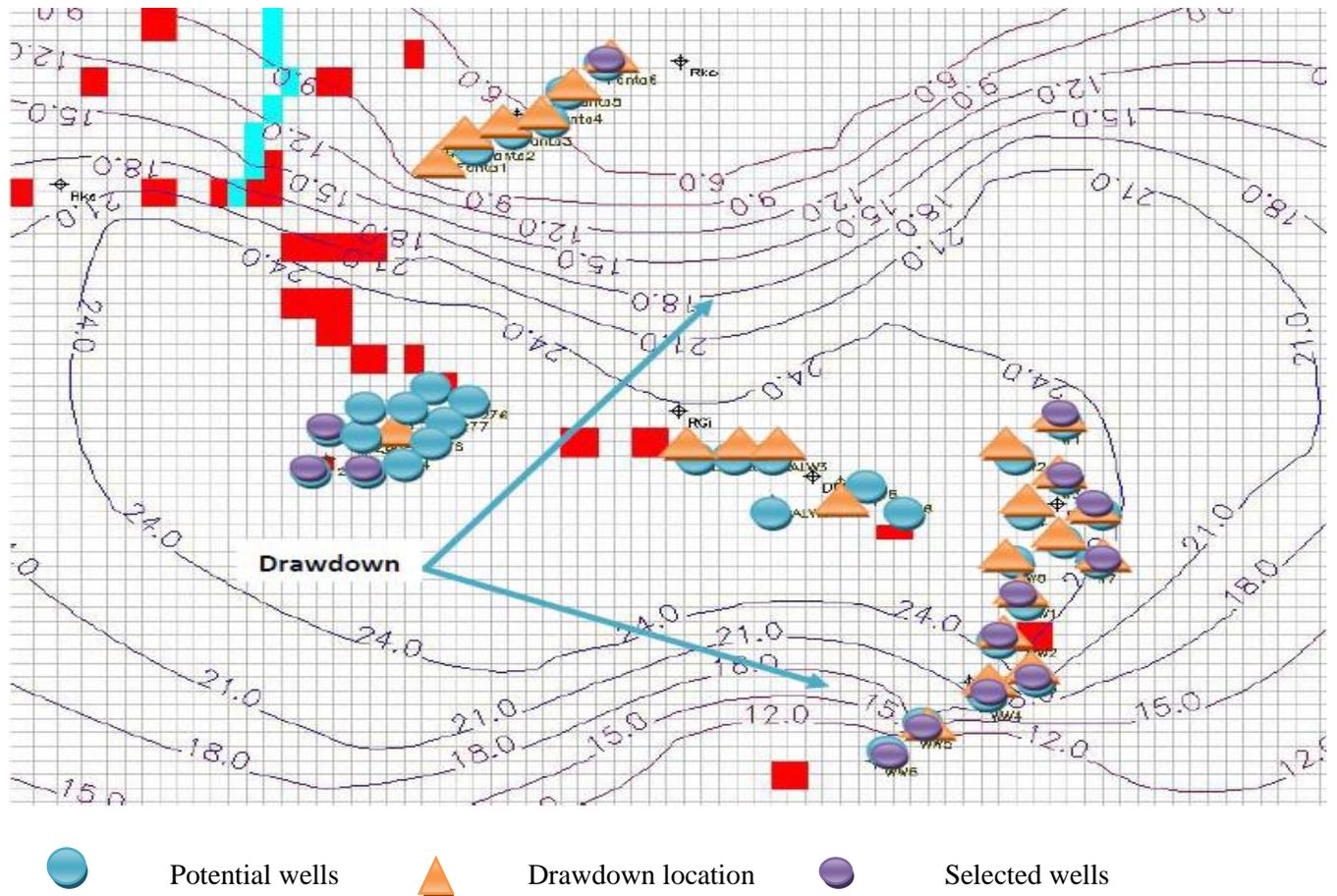


Figure 4-11 Drawdown and selected wells for 25 m drawdown constraint_case3

Table 4-14 Groundwater balance for 25 m drawdown constraint of case3 (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,228.8	24,451	5,270	64627.2	281,578
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 25 m drawdown	281,059.2	570.2	281,629	169,879.7	52,082	2609	57,084.5	281,655
	99.8%	0.2%	100%	60.3%	18.5%	0.92%	20.3%	100%
Difference (m ³ /day)	0	51.8	51	-17,349	27,631	-2661	-7543	77

The water balance (table4-14) shows that: 60.3% recharge is discharged by constant head boundary, 18.5% by wells, 0.92% by drains and 20.3% by river flow. When compared with the water balance of original steady state, the result shows increase of abstraction rate by 27,631 m³/day has caused decrease of outflow to river by 7543 m³/day, outflow to drains (springs) by 2661 m³/day and outflow to constant head boundary 17,349 m³/day.

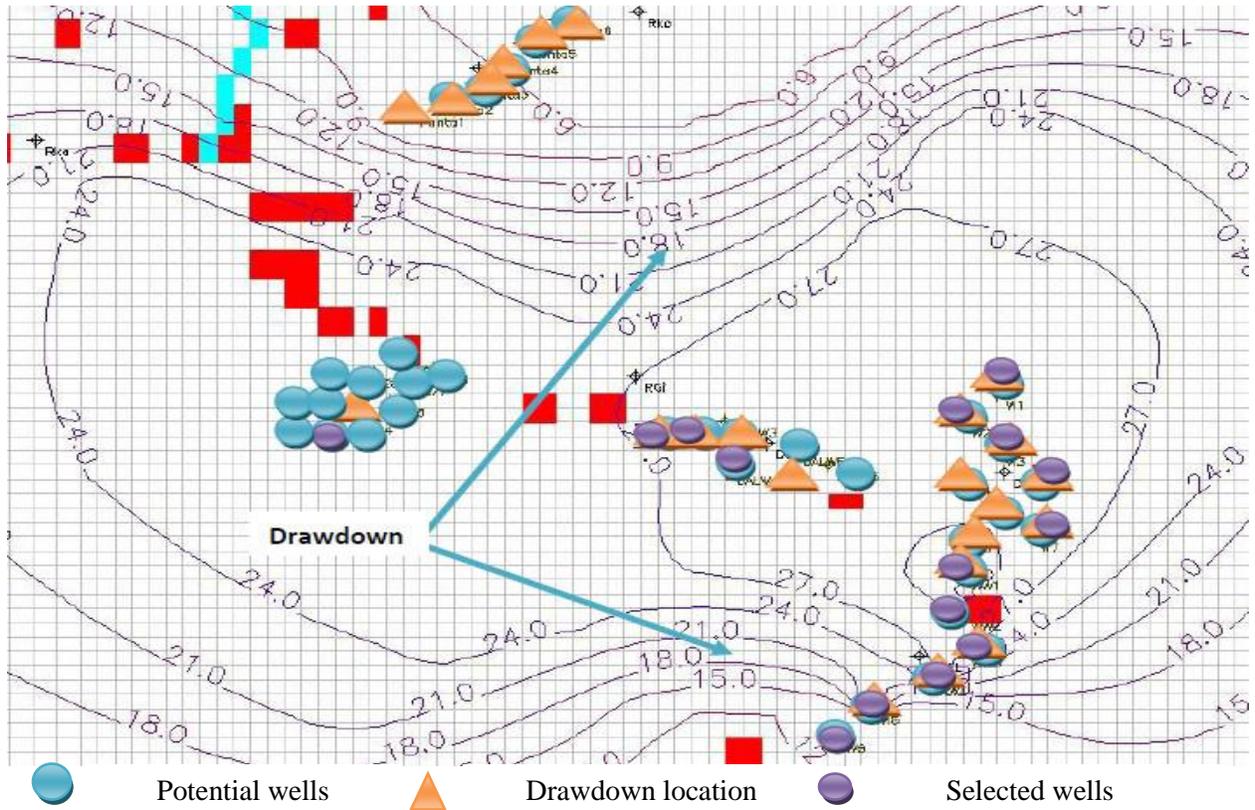


Figure 4-12 Drawdown and selected wells for 30 m drawdown constraint_case3

The water balance (table4-15) shows that: 59.3% of recharge is discharged by the constant head, 19.4% by wells, 0.92% by drains and 20.2% by river flow. When compared with water balance of original steady state, the result shows increase of abstraction rate by 30,180 m³/day has caused decrease of outflow to rivers by 7733 m³/day, outflow to drains (springs) by 2678 m³/day and outflow to constant head boundary by 20,287 m³/day.

Table 4-15 Groundwater balance for 30 m drawdown constraint of case3 (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,229	24,451	5,270	64627	281,578
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 30 m drawdown	281,059.2	570.2	281,629	166,942.1	54,631	2592.0	56,894	281,059
	99.8%	0.2%	100%	59.3%	19.4%	0.92%	20.2%	100%
Difference (m ³ /day)	0	51.8	51	-20,287	30,180	-2678	-7733	519

4.1.3 Results of cost minimization

The objective function of cost for all wells in well fields is obtained by considering the drilling cost of wells, cost of pipeline from wells to reservoir and operational cost of wells. The cost for pipe distribution line is obtained by assuming linear distance between wells and reservoir location. Additionally, balance constraints were introduced, for the four different drawdown constraint values (15, 20, 25 and 30 m) that guarantee a minimum of certain total abstraction rate. All these formulations were already introduced in section 4.1.1. Following the analysis carried out when optimizing the abstraction rate, in cost minimization drawdown constraints are imposed according to case three: for those wells or well fields in lower transmissivity zone drawdown constraints are imposed at each well, whereas for high transmissivity zones drawdown constraint is applied at centre of well fields. Table 4-16 shows optimal abstraction rates, total costs in ETB and selected wells during minimization of cost.

These results show that, this optimization approach is better compared to just maximization of total abstraction rates. With balance constraints that are very close to the maximum abstraction rates obtained from the previous optimization, the well selection is in fact much better. Fewer

wells are selected for all drawdown constraint values, and all selected wells are with significant pumping rates. There are virtually no wells with very low pumping rates (except few for constraints of 15 m). For sake of comparison the costs obtained for drawdown constraint of 15, 20, 25, and 30 m in previous optimization (28, 28, 29 and 28 million ETB) respectively (when only total abstraction rate was maximized) are compared to the costs obtained from this optimization for the same drawdown constraint (22.5, 12.6, 14.1 and 13.3 million ETB) respectively. It is obvious that for nearly same abstraction rate this optimization approach gives much better well configuration and consequently lower total cost.

The results also show that for higher draw downs fewer and generally different wells are selected in the optimal solution. Given the high contribution of installation costs (drilling and pipeline installation), this leads to high costs for drawdown of 15 m, compared to the costs obtained for higher draw downs.

Table 4-16 Optimal abstraction rate for cost minimization

Selected wells	Optimal abstraction rates(m ³ /day)			
	Drawdown imposed at each centre location (m)			
	15m	20m	25m	30m
Akaki_276	-4320.0	-4320.0	-4320.0	-3058.6
Akaki_277	-1762.6	---	-1589.8	---
Akaki_279	---	---	-4320.0	---
Fanta_6	-3957.1	-2713.0	---	---
Dalota_1	---	---	---	-2730.2
Dalota_4	---	-4320.0	-3309.1	-4320.0
Dalota_5	---	-1658.9	---	-4320.0
Dukem up_1	-846.7	---	---	---
Dukem up_5	-915.8	---	-2445.1	---
Dukem up_7	-1010.9	---	---	-3577.0
Dukem down_1	-1987.2	-4320	-4320.0	-4320.0
Dukem down_3	-181.4	---	---	---
Dukem down_4	-345.6	---	---	---
Dukem down_5	-1097.3	-1676.2	-2168.6	-2730.2
Dukem down_6	-4320.0	-4320	-4320.0	-4320.0
Total abstraction(m ³ /day)	-20,736.0	-23,328.0	-26,784.0	-29,376.0
Total cost (Million ETB)	22.5	12.6	14.1	13.3

Figures 4-13 to 4-16 present the optimal solutions for this cost optimization using draw downs of 15, 20, 25 and 30 m respectively, with the same symbols as in the first optimization reported in 4.1.2. Tables 4-17 to 4-20, present the water balance results from the optimal solutions for cost minimization.

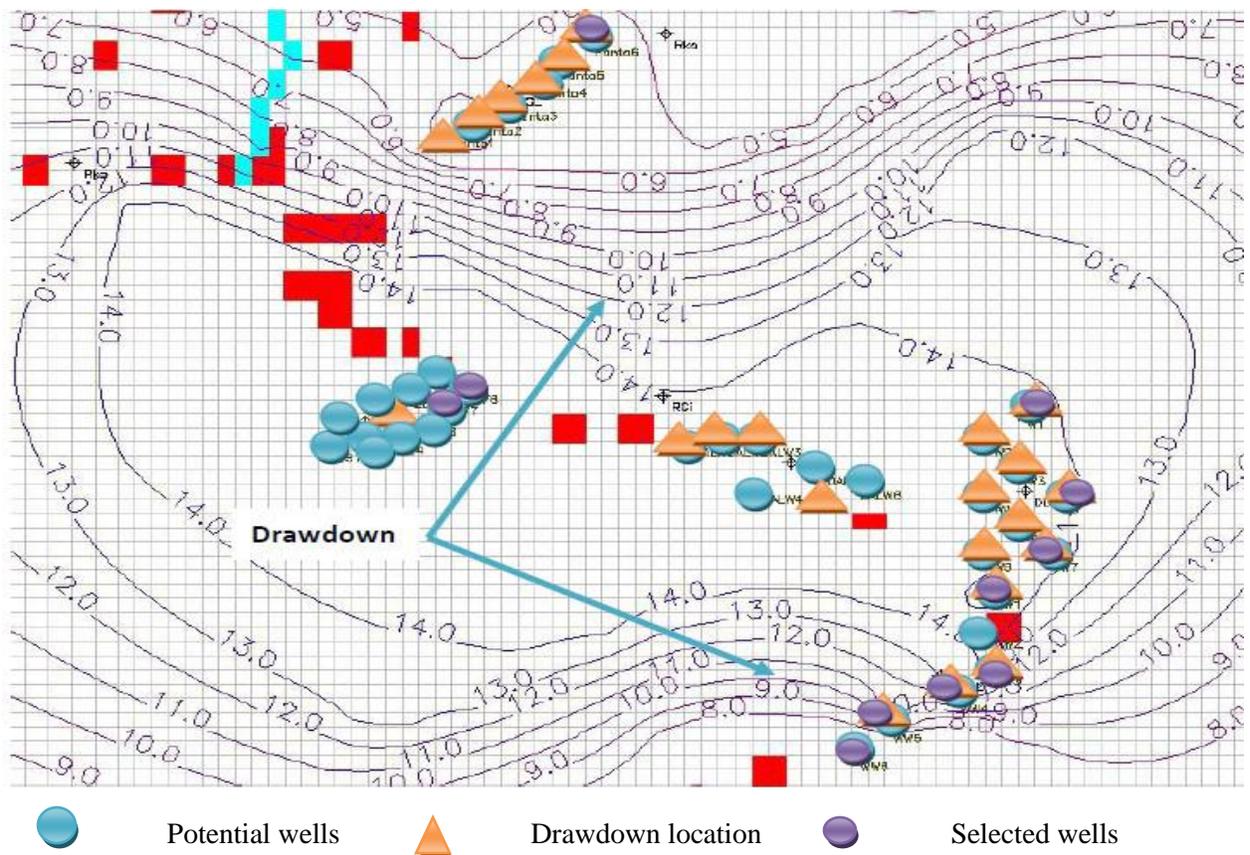


Figure 4-13 Drawdown and selected wells for 15 m drawdown constraint of cost function

Table 4-17 Groundwater balance for 15 m drawdown constraint of cost function (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,228.8	24,451	5,270	64627	281,577.6
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 15 m drawdown	281,059.2	561.6	281,621	175,962.2	42,993	3473	59,219	281,646.7
	99.8%	0.2%	100%	62.5%	15.3%	1.2%	21.0%	100%
Difference (m ³ /day)	0	43.2	43	-11,267	18,541	-1797	-5409	69.1

The water balance (table4-17) shows that: 62.5% of recharge is discharged by constant head boundary, 15.3% by wells, 1.2% by drains (springs) and 21.0% by river flow. When compared with water balance of original steady state, the result shows increase of abstraction rate by 11,267 m³/day has caused decrease of outflow to river by 5409 m³/day, outflow to drains (springs) by 1797 m³/day and outflow to constant head boundary by 11,267 m³/day.

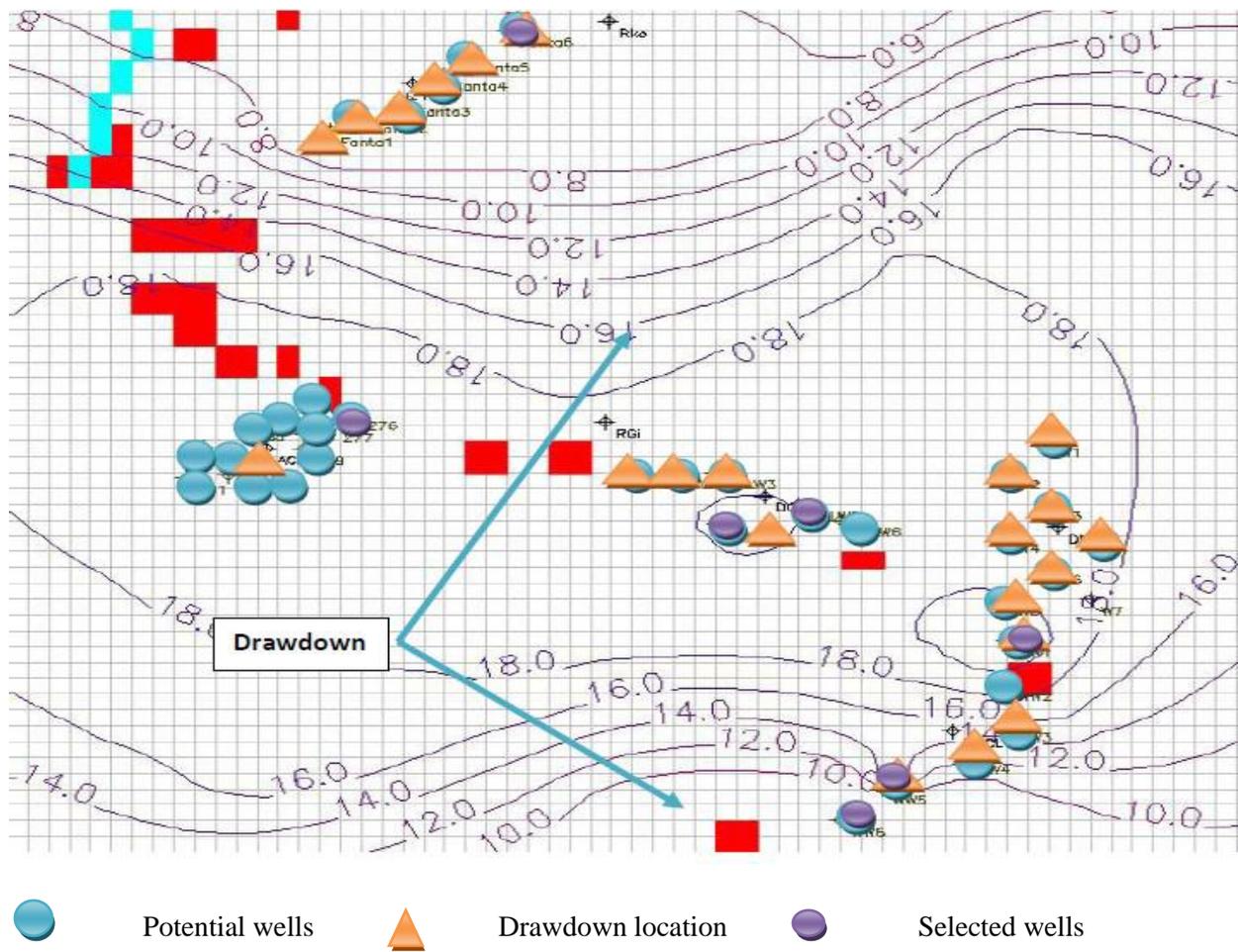


Figure 4-14 Drawdown and selected wells for 20 m drawdown constraint of cost function

Table 4-18 Groundwater balance for 20 m drawdown constraint of cost function (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,228.8	24,451.2	5,270.4	64627	281,578
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 20 m drawdown	281,059.2	570.2	281,629	173,508.5	47,787.8	2911.7	57,456	281,664
	99.8%	0.2%	100%	61.6%	17.0%	1.0%	20.4%	100%
Difference (m ³ /day)	0	51.8	51	-13,720	23,337	-2359	-7171	86

The water balance (table4-18) shows that: 61.6% of recharge is discharged by constant head boundary, 17% by wells, 1% by drains and 20.4% by river flow. When compared with water balance of original steady state model, the result shows that increase of abstraction by 23,337 m³/day has caused decrease of outflow to river by 7171 m³/day, outflow to drains (springs) by 359 m³/day and outflow to constant head boundary by 13,720 m³/day.

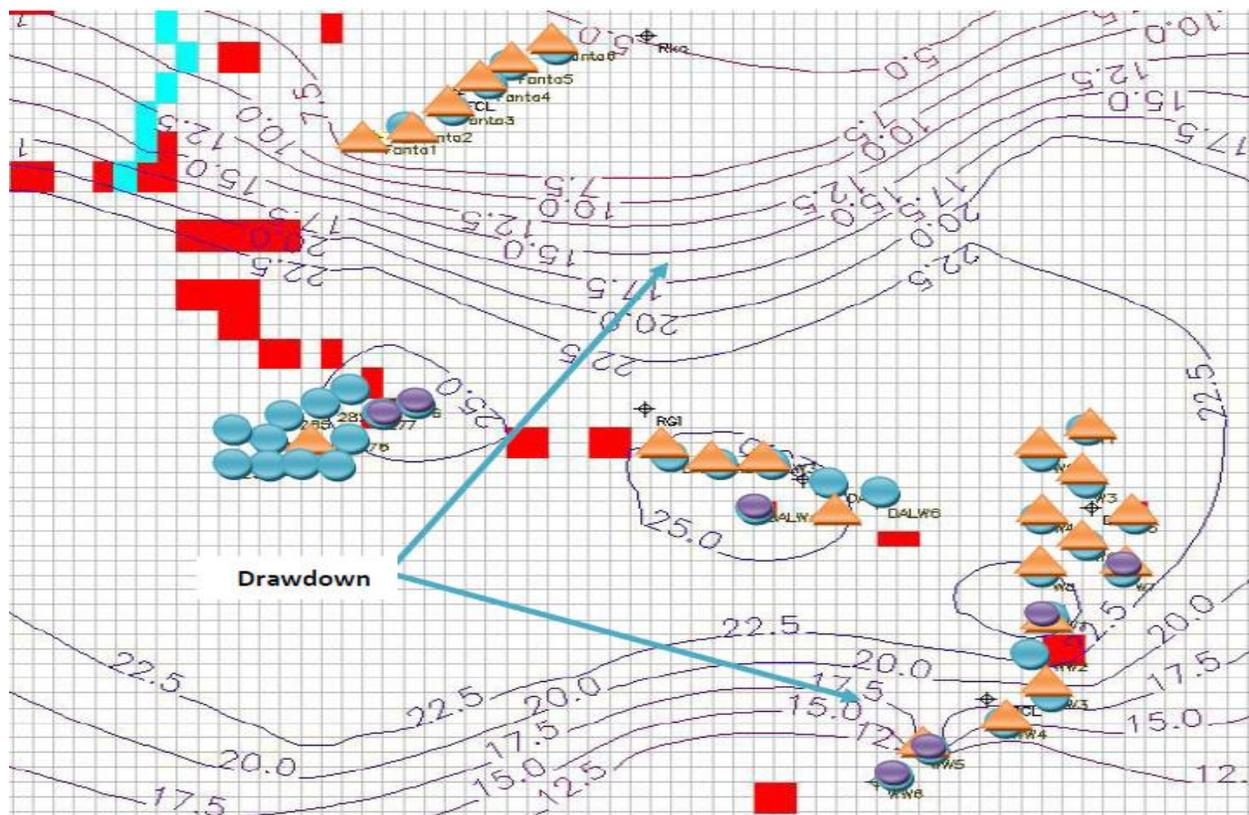


Figure 4-15 Drawdown and selected wells for 25 m drawdown constraint of cost function

Table 4-19 Groundwater balance for 25 m drawdown constraint of cost function (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,229	24,451	5,270	64,627	281,578
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 25 m drawdown	281,059	570.2	281,629	170,424	51,244	2,678	57,318	281,664
	99.8%	0.2%	100%	60.5%	18.2%	0.95%	20.4%	100%
Difference (m ³ /day)	0	51.8	51	-16,805	26,793	-2,592	-7,309	86

The water balance (table 4-19) shows that: 60.5% of recharge is discharged by constant head boundary, 18.2% by wells, and 0.95% by drains (springs) and 20.4% by river flow. When compared with water balance of original steady state model, the result shows that the increase of abstraction rate by 26,793 m³/day has caused decrease of outflow to river by 7309 m³/day, outflow to drains (springs) by 2592 m³/day and outflow to constant head boundary by 16,805 m³/day.

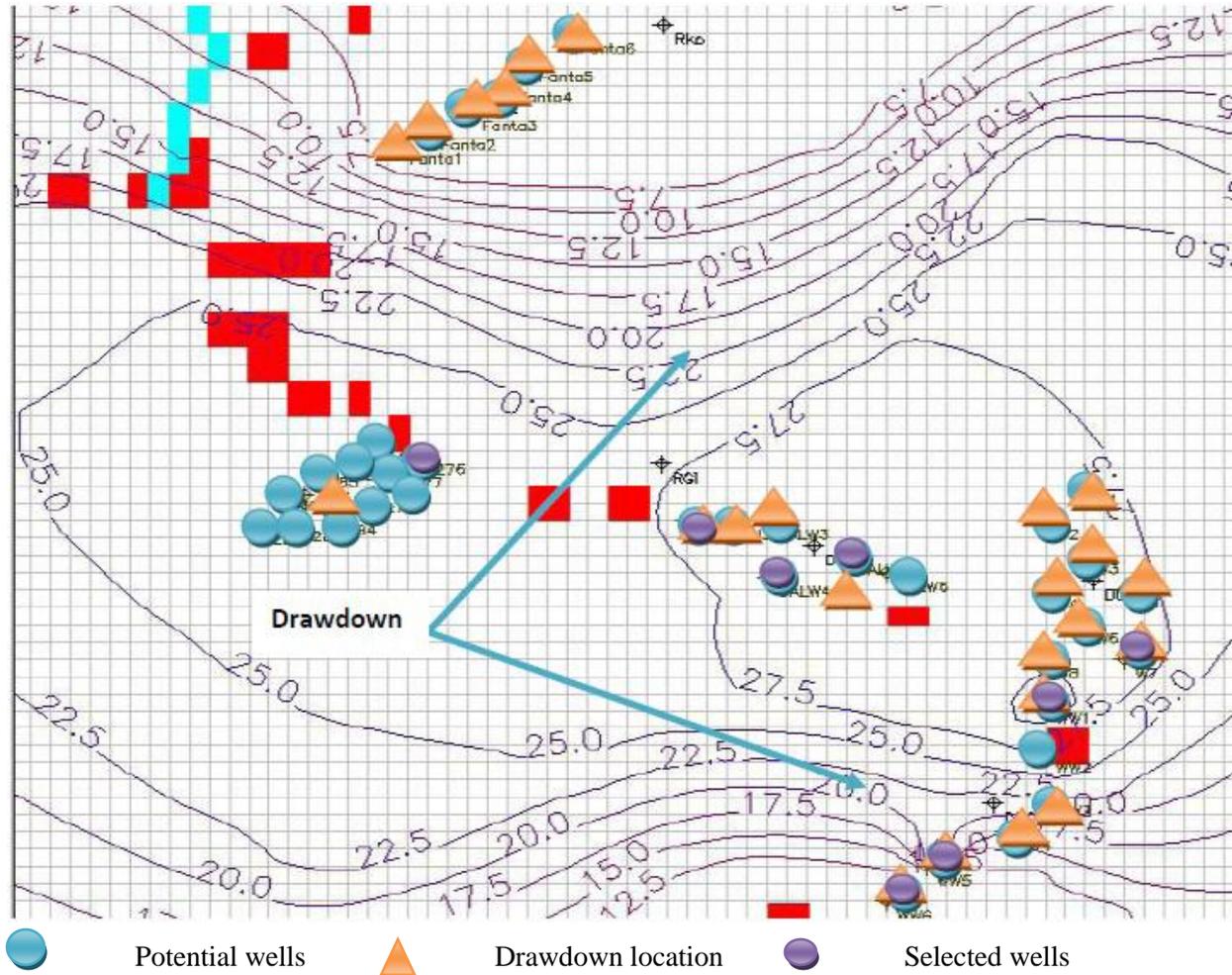


Figure 4-16 Drawdown and selected wells for 30 m drawdown constraint of cost function

Table 4-20 Groundwater balance for 30 m drawdown constraint of cost function (m³/day)

	Inflow to catchment			Outflow from catchment				
	Natural recharge	River recharge	Total	Constant head boundary	Wells	Drains	River flow	Total
Original steady state water balance	281,059.2	518.4	281,578	187,229	24,451	5,270.4	64627	281,577.6
	99.8%	0.2%	100%	66.5%	8.7%	1.9%	23%	100%
Water balance of 30 m drawdown	281,059.2	570.2	281,629	168,290	53,836	2592.0	56,946	281,664.0
	99.8%	0.2%	10%	59.8%	19.1%	0.92%	20.2%	100%
Difference (m ³ /day)	0	51.8	51	-18,939	29,385	-2678	-7681	86.4

The water balance (table 4-20) shows that: 59.8% of recharge is discharged by constant head boundary, 19.1% by wells, 0.92% by drains (springs) and 20.2% by river flow. When compared with water balance of original steady state model, the result shows that increase of abstraction rate by 29,385 m³/day has caused decrease of outflow to rivers by 7681 m³/day, outflow to drains (springs) by 2678 m³/day and outflow to constant head boundary by 18,939 m³/day.

4.2 Unsteady state condition

4.2.1 Problem formulation

The objective function of transient state condition is formulated for three stress periods with equal time steps of 10 years. This formulation depends on planning horizon of AAWSSA. The drawdown constraints are applied at 23 control locations. These control locations are same as case three, developed during steady state analysis. In the first stress period 15 m drawdown constraint is applied. In the second stress period 20 m drawdown constraint is applied whereas in the third stress period 25 m drawdown constraint is applied. The selection of drawdown constraint values is done depending on the operational cost of wells and effects of drawdown to the city. In transient condition two similar objectives are set as in the steady state condition. The mathematical formulation of the objectives is presented below.

Objective one: Maximization of groundwater abstraction at Akaki, Fanta, Dalota, Dukem up and Dukem down well fields in transient case. The mathematical expression for management problem is formulated as below.

$$\text{Max: } Z = \sum_{k=1}^T \sum_{j=1}^N Q_j \quad 4-9$$

Subject to constraints:

$$DM = U_i - H_i = \sum_{k=1}^T \sum_{j=1}^N R_{ij} \cdot Q_j \quad 4-10$$

$$DM_l \leq DM \leq DM_u \quad 4-11$$

$$0 \leq Q_j \leq Q_{jmax} \quad 4-12$$

Where:

Q_j is rate at managed well location j (negative for pumping)

U_i is unmanaged head

H_i is managed head

DM_l is lower drawdown limit at each control location, =0.0001

DM_u is maximum drawdown limit at each control location = 15, 20, 25 m for each stress period

Q_{jmax} is the maximum abstraction rate for each of managed wells = 4320 m³/day

$i=1, 2, 3... N$ is control point location

$j=1, 2, 3... N$ is pumping wells

$k= 1, 2, 3... T$ is number of stress period

Objective two: Minimizing the cost of well system to obtain optimal water abstraction rate and drawdown at certain level. The formulation of objective function of cost and the mathematical expression is the same as the steady state case. Therefore, the mathematical formulation is referred to steady state condition mentioned above in section 4.1.1. The additional thing to steady state condition is that balance constraints are now applied for each stress period. The balance constraint applied for stress period one, stress period two and stress period three is 50,976m³/day, 40,608m³/day and 42,336m³/day respectively. These balance constrain values are same as total abstraction rates obtained from maximization of abstraction rate (objective one) for respective stress period. Besides, a special specification of integer constraints is imposed to keep one already selected well in one stress period to remain active for the next stress period. Similarly to steady state case the operational cost is included via the MODMAN pre-processor, but the drilling and pipe costs are not created directly in MODMAN. In order to include these costs integer constraints have to be introduced and modified manually in the MPS file.

4.2.2 Results of maximization of abstraction rate

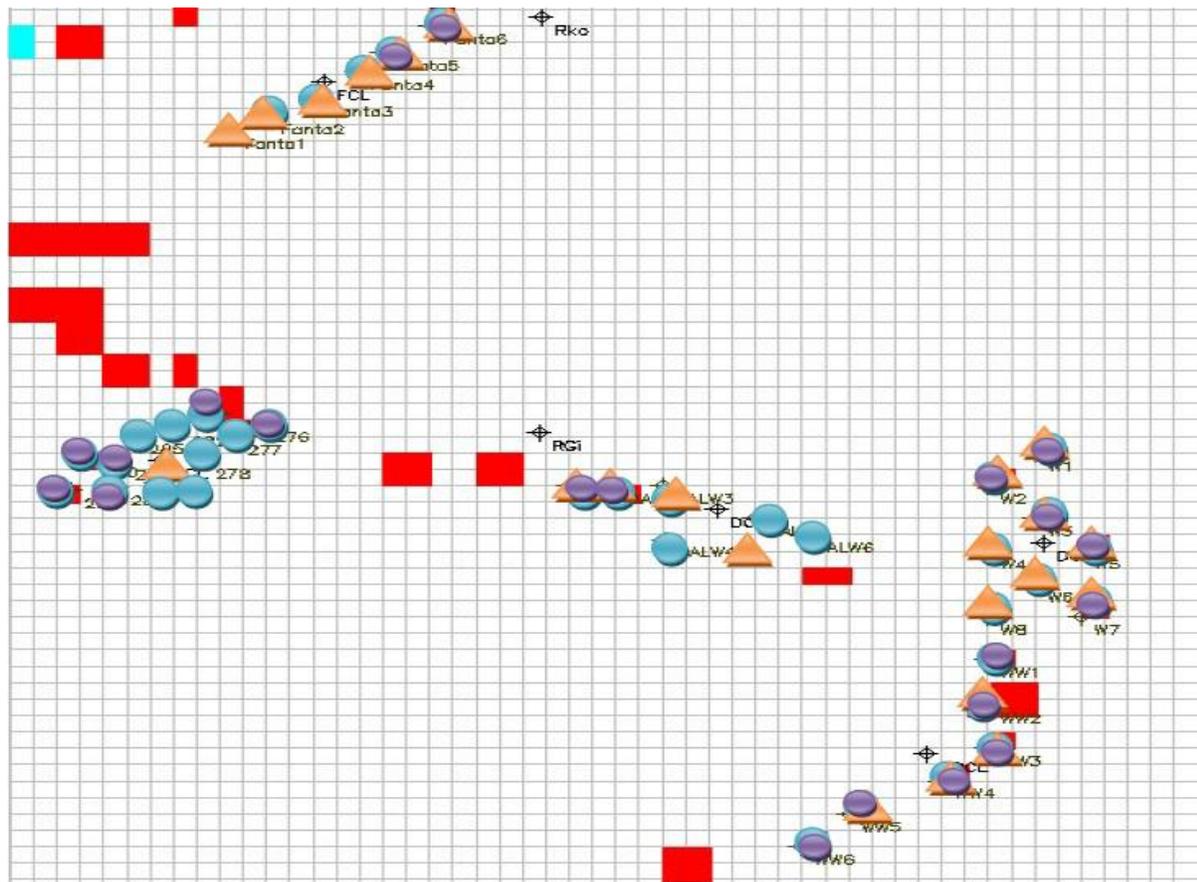
In transient state case, in order to obtain maximum abstraction rate from the five well fields, drawdown constraints of 15 m, 20 m and 25 m are imposed on control location for three equal stress periods of 10years respectively. The selected wells, optimal abstraction rates for each stress period are presented below in Table 4-21.

The result show generally higher total abstraction compared to those calculated from steady state conditions. The main reason for this is the contribution from aquifer storage, as subsequent results demonstrate.

Table 4-21 Selected wells and optimal abstraction rates in transient condition

Selected wells	Optimal abstraction rates (m ³ /day)		
	Stress periods		
	Period_1	Period_2	Period_3
Akaki_276	-4320	-4320	-4320
Akaki_279	-2196.6	0	-1467.8
Akaki_286	-4320	-420.4	0
Akaki_287	-4320	-4320	-4320
Akaki_290	-4320	-4320	-4320
Akaki_291	-4320	-4320	-4320
Fanta_5	-2189.4	0	0
Fanta_6	-4320	-3574.5	-1418.8
Dalota_1	-697.3	-312.3	-114.2
Dalota_2	-619.1	-362.8	0
Dalota_3	0	-595.3	-25.9
Dalota_4	0	0	-3117.4
Dukem up_1	-2227.2	-1849.8	-1914.9
Dukem up_2	-1663.0	-1121.1	-748.9
Dukem up_3	-1138.2	-931.3	-893.2
Dukem up_5	-1816.8	-1740.6	-1934.3
Dukem up_7	-1761.4	-1945.1	-2338.4
Dukem down_1	-4320	-3670.08	-3467.4
Dukem down_2	-69.6	-113.9	-176.4
Dukem down_3	-271.0	-321.1	-402.6
Dukem down_4	-507.9	-601.4	-742.6
Dukem down_5	-1477.8	-1840.7	-2299.1
Dukem down_6	-4320	-4320	-4320
Total abstraction rate (m ³ /day)	-51,195.3	-41,000.2	-42,661.7
Total cost (Million ETB)	39.3	1.03	1.19

Figures 4-17 to 4-19 present the optimal solution in terms of selected wells in each stress period.



Potential wells
 Drawdown location
 Selected wells

Figure 4-17 Selected wells in stress period 1 of transient state condition

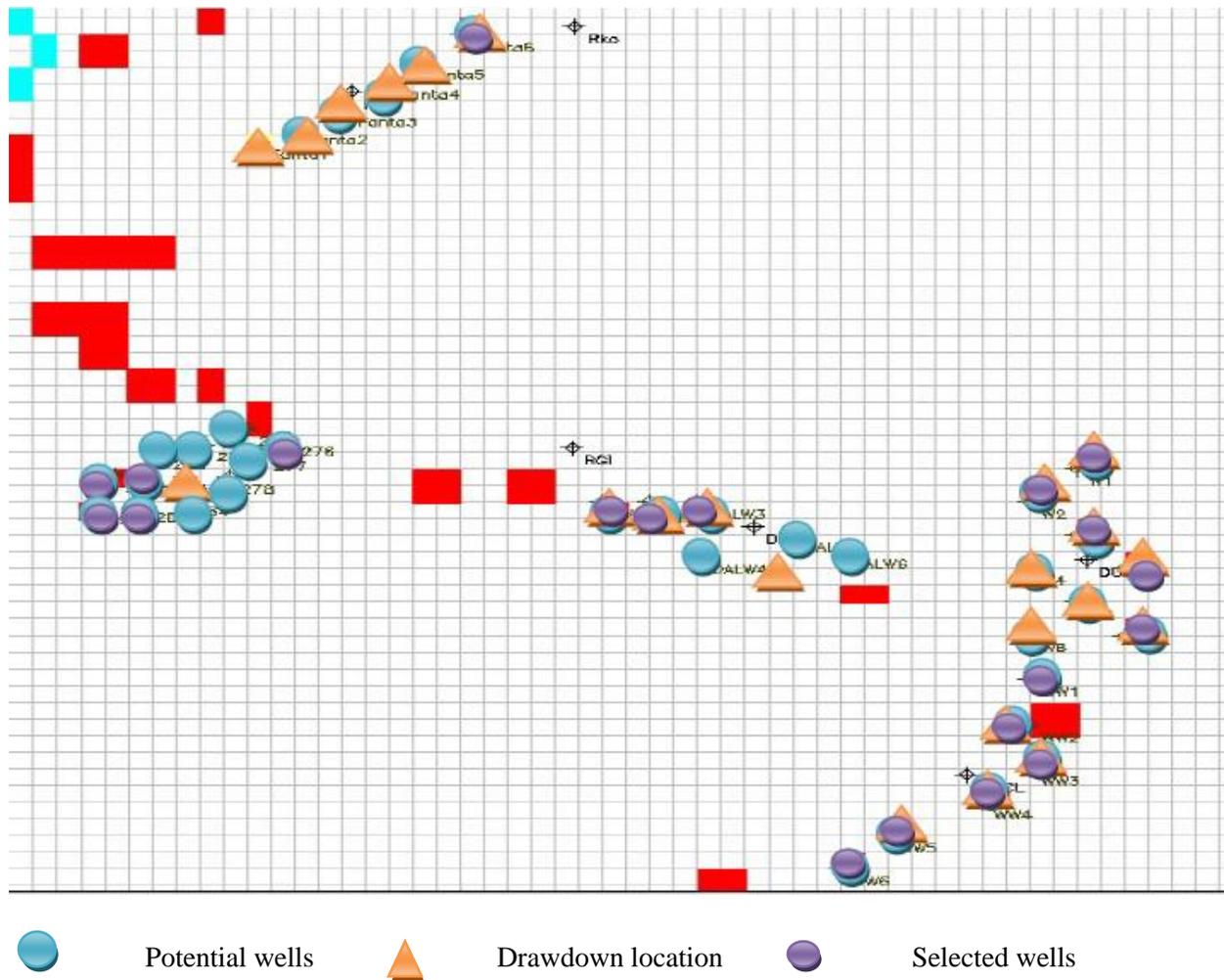


Figure 4-18 Selected wells in stress period 2 of transient state condition

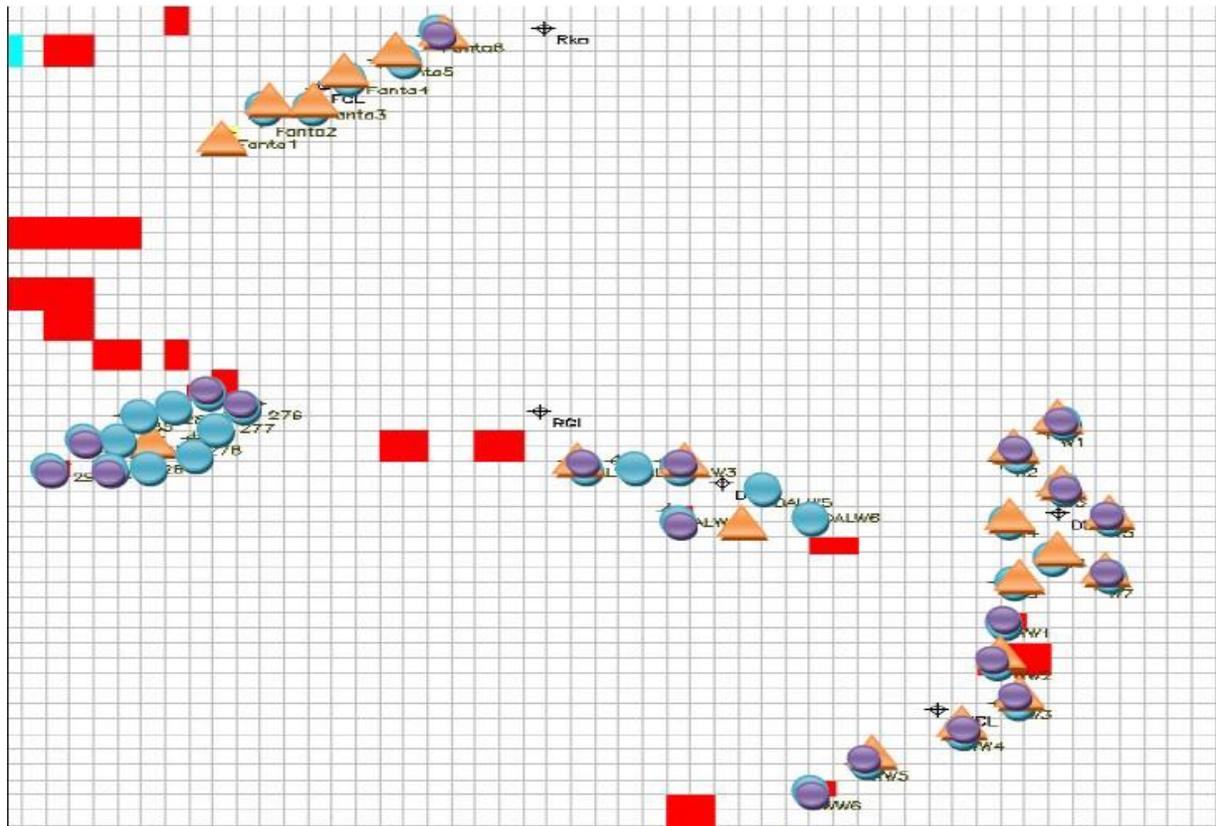


Figure 4-19 Selected wells in stress period 3 of transient state condition

Figure 4-20 below shows the drawdown development in time for the five well fields (average of all wells per well fields) after MODFLOW simulation with the optimal abstraction rates. It shows that the calculated drawdown values at each well field are smaller than the specified drawdown constraint during optimization. Therefore, from this point of view the obtained optimal abstraction rates are acceptable.

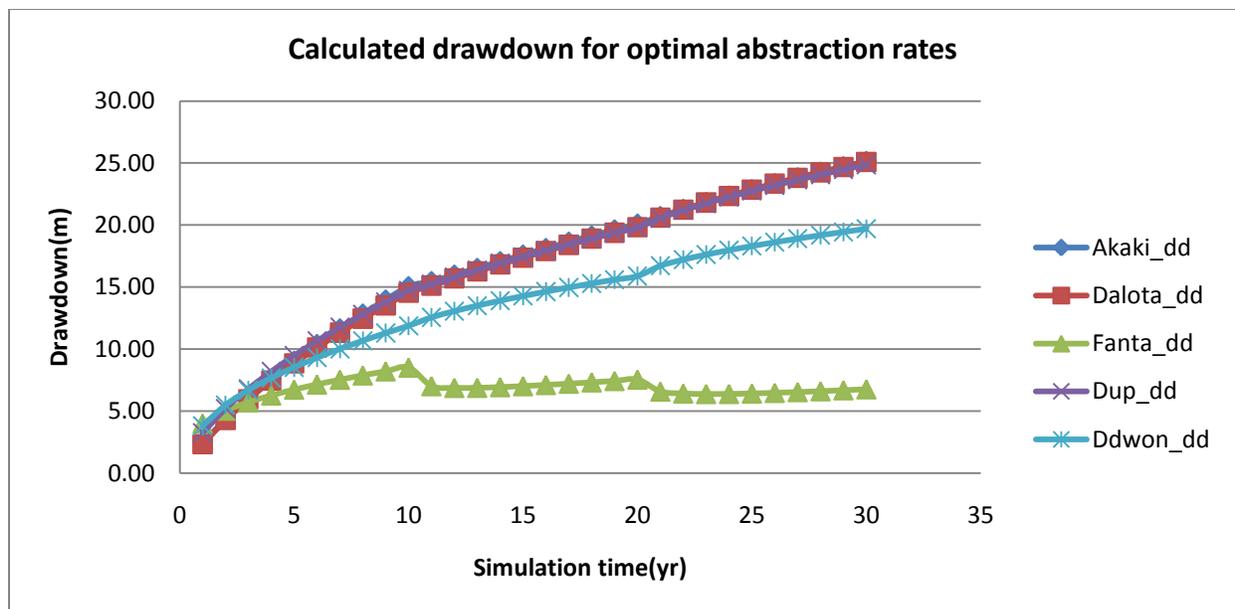


Figure 4-20 Calculated drawdown for maximization of abstraction rate in transient case

In order to provide an analysis of the changes in the water balance with the optimal solution, first the water balance of the original transient model (Table 4-22) is presented. Similarly to the steady state case it shows that most of the inflow comes from natural recharge in all stress periods. For the outflow terms, 187,315.2 m³/day (66.5%) m³/day recharge is discharged by constant head boundary, 24,451.2 m³/day (8.7%) is discharged by wells, 5313.6 m³/day (1.9%) is discharged by drains (springs) and 64,782.7 m³/day (23%) is discharged by river. The percentage (%) shows percentage of each components of water balance with respect to the total inflow to the catchment.

Table 4-22 Water balance of original transient model (m³/day)

Stress period	Inflow to catchment				Outflow from catchment				
	Natural recharge	River recharge	Storage	Total	Constant head boundary	Wells	Drains	River flow	Total
1	281059.2	553.0	216	281828.2	187315.2	24451.2	5313.6	64782.7	281863
	99.8%	0.2%	0.08%	99.9%	66.5%	8.7%	1.9%	23.0%	100.0%
2	281059.2	553.0	152.1	281764.3	187315.2	24451.2	5305.0	64739.5	281811
	99.8%	0.2%	0.05%	99.9%	66.5%	8.7%	1.9%	23.0%	100.0%
3	281059.2	553.0	24.2	281636.4	187315.2	24451.2	4942.1	64869.1	281690
	99.8	0.2	0.009	100%	66.5%	8.7%	1.8%	23.0%	100.0%

Table 4-23 shows that after introducing the optimal abstraction rates at each of the selected wells in well fields, the water balance of the system is changed. In order to have additional pumping (abstraction) from the well fields, additional inflow needs to be induced. The water balance shows that storage inflow is significantly increased to balance the additional pumping. Additional balance for the increased pumping comes from reduction in outflow to constant head, rivers and drains (in that order of significance). There is no additional inflow component to balance additional discharge from the well fields.

In Table 4-23 the percentage (%) shows percentage of each component of water balance with respect to the total inflow (or outflow) to the aquifer system. The difference (m^3/day) given in the last three rows of the table shows, the difference between each components of water balance for each stress period with respect to the original water balance.

Table 4-23 Water balance of transient model after additional wells (m^3/day)

Type of water balance	Stress periods	Inflow to catchment				Outflow from catchment				
		Natural recharge	River recharge	Storage	Total	Constant head boundary	Wells	Drains	River flow	Total
Original water balance	1	281059	553	216	281828	187315	24451	5314	64783	281863
	2	281059	553	152.1	281764	187315	24451	5305	64740	281811
	3	281059	553	24.2	281636	187315	24451	4942	64869	281690
Water balance after additional wells	1	281059	553	40988	322600	182494	75653	3681	60800	322627
		87.1%	0.17%	12.7%	100%	56.6%	23.5%	1.14%	18.9%	100%
	2	281059	553	25298	306910	178468	65457	3444	59567	306935
		91.6%	0.18%	8.2%	100%	58.2%	21.3%	1.1%	19.4%	100%
	3	281059	561.6	22777	304398	175185	67119	2842.6	59277	304423
		92.3%	0.18%	7.5%	100%	57.6%	22.0%	0.9%	19.5%	100%
Difference (m^3/day)	1	0	0	40772	40772	-4821	51202	-1633	-3983	40764
	2	0	0	25146	25146	-8847	41005	-1861	-5173	25124
	3	0	8.6	22753	22762	-12131	42668	-2100	-5592	22733

4.2.3 Results of cost minimization

Table 4-24 shows the selected wells and their optimal abstraction rate during cost minimization. The result shows that wells that are selected in one stress period are kept to be selected in other stress period. Many wells are in fact selected during the first stress period, which brings high installation costs, so the costs presented here are much higher compared to the costs obtained for steady state case.

Table 4-24 Selected wells and abstraction rates of cost minimization-transient

Selected wells	Optimal abstraction rate(m ³ /day)		
	Stress periods		
	Period_1	Period_2	Period_3
Akaki_276	-4320	0	-4320
Akaki_277	-3323.2	-4240.9	-4320
Akaki_278	-2381.1	-4320	0
Akaki_279	-4320	0	-1863.8
Akaki_287	-4320	-4320	-4320
Akaki_291	-4320	-4320	-4320
Fanta_5	-2195.4	0	0
Fanta_6	-4320	-3606	-1429.1
Dalota_4	-4320	0	-402.7
Dalota_5	1555.5	-4254.4	-4320
Dukem up_1	-2696.5	-2305.6	-2636.3
Dukem up_7	-2350.9	-2489.4	-2624.6
Dukem down_1	-4188.5	-3917.7	-4320
Dukem down_4	-559.4	-662.2	-822.5
Dukem down_5	-1485.5	-1851.9	-2317.1
Dukem down_6	-4320	-4320	-4320
Total abstraction rate(m ³ /day)	-50,976	-40,608	-42,336
Total cost (Million ETB)	29	0.124	0.129

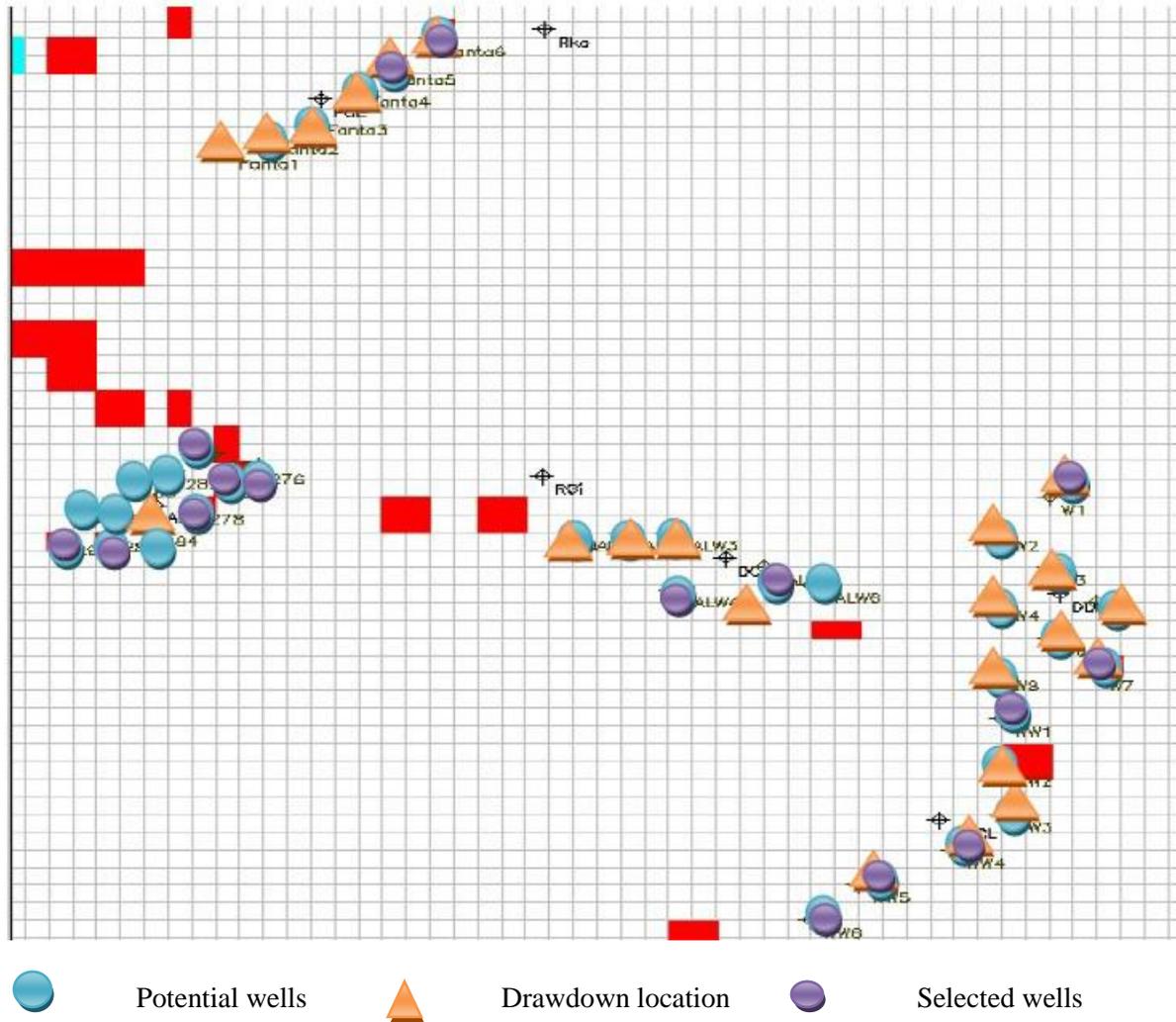


Figure 4-21 Selected wells of cost minimization in stress period one

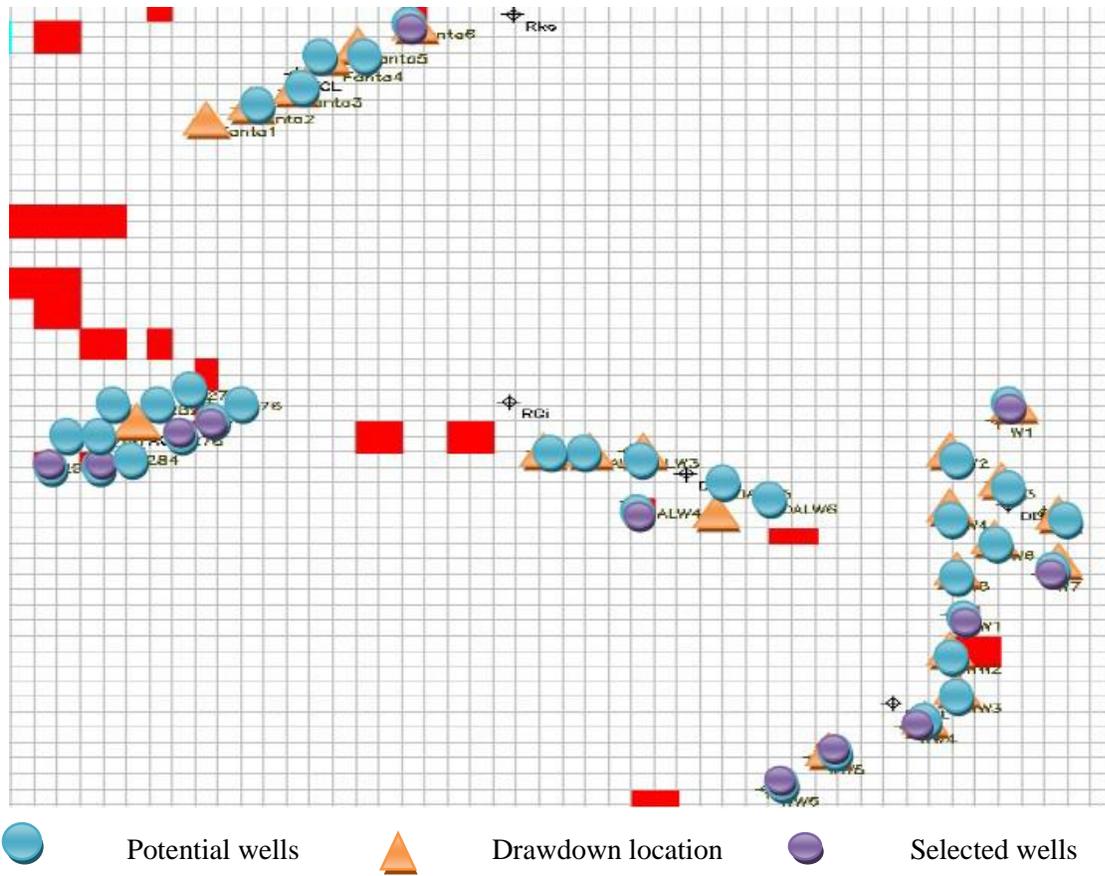


Figure 4-22 Selected wells of cost minimization in stress period two

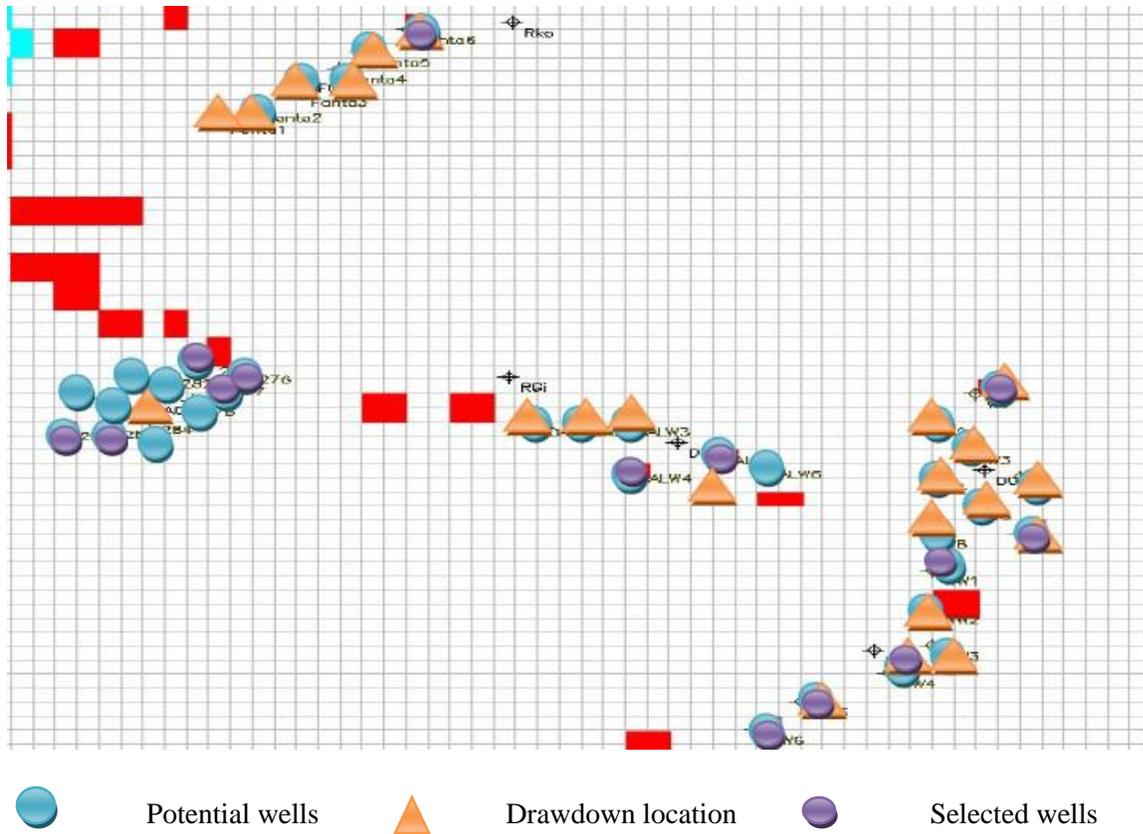


Figure 4-23 Selected wells of cost minimization in stress period three

The drawdown of each well field after introducing the optimal abstraction rate of each selected wells shows that there is no negative effect up on the drawdown constraints that are used during cost optimization. The calculated drawdown of each well field for the optimal abstraction rates of three stress periods is shown below.

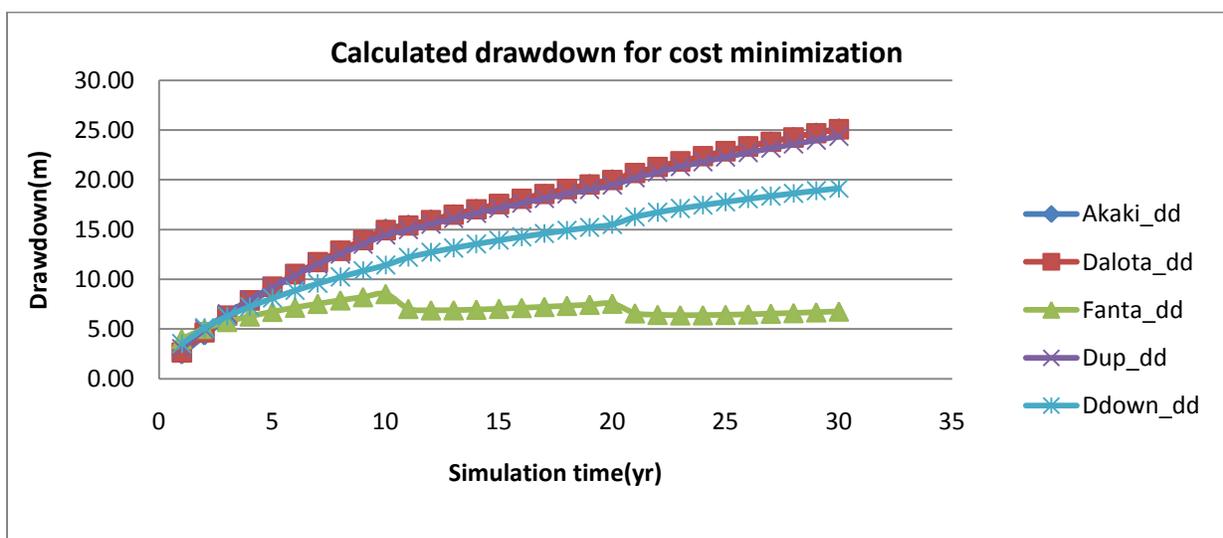


Figure 4-24 Calculated drawdown for cost minimization

Table 4-25 shows that after introducing the optimal abstraction rates at each of the selected wells in well fields, the water balance of the system is changed. In order to have additional pumping (abstraction) from the well fields, additional inflow needs to be induced. The water balance shows that storage inflow is significantly increased to balance the additional pumping. Additional balance for the increased pumping comes from reduction in outflow to constant head, rivers and drains (in that order of significance). There is no additional inflow component to balance additional discharge from the well fields.

In Table 4-25 the percentage (%) shows percentage of each component of water balance with respect to the total inflow (or outflow) to the aquifer system. The difference (m³/day) given in the last three rows of the table shows, the difference between each components of water balance for each stress period with respect to the original water balance.

Table 4-25 Water balance after additional wells- transient

Type of water balance	Stress periods	Inflow to catchment				Outflow from catchment				
		Natural recharge	River recharge	Storage	Total	Constant head boundary	Wells	Drains	River flow	Total
Original water balance	1	281059	553	216	281828	187315	24451	5314	64783	281863
	2	281059	553	152.1	281764	187315	24451	5305	64740	281811
	3	281059	553	24.2	281636	187315	24451	4942	64869	281690
Water balance after additional wells	1	281059	553	40826	322438	182548.5	75436	3680.6	60799.7	322466
		87.2%	0.17%	12.7%	100%	56.6%	23.4%	1.1%	18.9%	100%
	2	281059	553	24996	306608	178554.2	65067.8	3447.4	59564.2	306634
		91.7%	0.2%	8.2%	100%	58.2%	21.2%	1.1%	19.4%	100%
	3	281059.2	562	22585	304206	175314.2	66796	2846	59279	304235
		92.4%	0.2%	7.4%	100%	57.6%	22.0%	0.94%	19.5%	100%
Difference (m ³ /day)	1	0	0	40610	40610	-4767	50985	-1633	-3983	40603
	2	0	0	24843	24844	-8761	40617	-1858	-5175	24823
	3	0	9	22561	22570	-12001	42345	-2096	-5590	22545

5 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

Groundwater management model MODMAN is used to link simulation model MODFLOW with optimization model LINDO to obtain the optimal abstraction rate and well location of Akaki catchment well fields in steady and transient state condition. The objectives of problem formulation are maximization of abstraction rate and minimization of cost in both steady and unsteady state conditions. The hydrological constraints are imposed to get reliable results of optimal abstraction rates and well locations for both objectives. Therefore, the obtained result of the model shows that simulation-optimization model can be used for optimization of abstraction rates in large catchment areas in both steady and transient state conditions.

With respect to the specific objectives set out in this study the following conclusions can be drawn:

1. During selection of control location to impose the drawdown constraints transmissivity of the control location is needs to be considered. The best way of specifying drawdown constraints is found to be as follows: For control locations in low transmissivity zones drawdown constraints are applied at each managed well whereas for locations in high transmissivity zones drawdown constraints are applied at centre location of the well fields.
2. When maximizing abstraction rates in steady state conditions, total abstraction rates between 20,000 and 30, 000 m³/day are obtained, depending on specified drawdown constraints (15m-30m)
3. When minimizing total costs (installation + operational costs) in steady state conditions, similar total abstraction rates are found, but with better well configurations that lead to smaller total costs. For larger drawdown constraints the numbers of chosen wells in optimal solutions are smaller (12-14 Million ETB), which leads to smaller total costs compared to smaller drawdown constraints (22 Million ETB). This is due to high installation costs per well.
4. In unsteady state optimization, for a period of 30 years, larger total abstraction rates are obtained, mainly due to supply of water for abstraction from aquifer storage. With drawdown constraints varying from 15 m in first 10 years, 20 m in second 10 years and 25 m in last 30 years, the total abstraction rate varies from ~ 50,000 m³/day in first period, ~40,000 m³/day in second stress period and ~43,000 m³/day in third stress period. Because of the condition of maintaining installed wells in second and third period if they are introduced in first period, and since in first period many wells are selected, the total costs in unsteady conditions are quite high (~29 Million ETB).

Further conclusions from this study are drawn as follows:

- ✓ During simulation of the obtained optimal abstraction rates of each well in regional groundwater flow model MODFLOW of the area, the optimal rates do not create any higher drawdown as compared to imposed drawdown constraints in both steady and unsteady state cases. In general as the drawdown constraint increases the abstraction rate also increases and this in turn increases total costs of wells. To limit this operational cost due to induced drawdown, a maximum acceptable drawdown is set in order to obtain significant amount of water from the well fields. Despite its high abstraction rate, drawdown of more than 25 meter has been found to cause high cost in the well field areas. Hence, maximum drawdown suggested within the well fields to obtain significant amount of abstraction from the well fields is 25 meter, which was also used as maximum drawdown after 30 years in the unsteady state simulations.
- ✓ The water balance of transient state condition shows significant amount of water available for extraction from the well fields, while meeting the imposed drawdown constraints. From definition of sustainable yield (Zhou, 2009) a sustainable yield as percentage of recharge allowed, it can be concluded that additional abstraction of groundwater from well fields do not have high too depletion of groundwater resources in the area of the well fields. Besides this, additional wells to well fields do not have significant impact to flows of river, constant head boundary and drains (springs).

5.2 Recommendations

Some of important recommendations made from this study are given below.

- The obtained optimization result of abstraction rate of wells highly depends on the existing regional groundwater model of Akaki areas. Therefore, in order to have more reliable results of abstraction rate, the regional groundwater model of Akaki catchment has to be built by considering the multilayer aquifer approach and with defined geometry of aquifer. An optimization study should then be repeated with the new, improved model
- Depending on the demand of water in the city and future plan of extraction from well fields, the well fields need to be used phase by phase starting from Akaki well field which has higher groundwater potential compared to the other well fields.
- In future study comparison of the result of the current simulation-optimization (MODFLOW, via MODMAN with LINDO) can be done with results of simulation coupled with global optimization, e.g. with genetic algorithm optimization (MODFLOW with GA).
- The number of wells selected in maximization of abstraction rate is higher than number of wells in minimization of costs. But the total amount of abstraction is almost same in both cases. Therefore, it is recommended to use wells that are selected by cost minimization.
- Uncertainties and model assumptions made have to be considered during using the obtained model results.

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Appendices

Appendix 1: Calculated drawdown for maximization of abstraction rate

Simulation time(yr)	Calculated drawdown(m)				
	Akaki	Dalota	Fanta	Dukem up	Dukem down
1	2.47	2.34	4.0	3.25	3.81
2	4.37	4.28	5.0	5.20	5.49
3	6.06	5.96	5.7	6.79	6.69
4	7.61	7.46	6.3	8.20	7.67
5	9.05	8.84	6.7	9.47	8.53
6	10.40	10.12	7.1	10.66	9.30
7	11.66	11.32	7.5	11.76	10.01
8	12.86	12.45	7.9	12.80	10.67
9	13.99	13.52	8.2	13.79	11.30
10	15.07	14.54	8.5	14.72	11.88
11	15.47	15.12	7.0	15.23	12.56
12	16.00	15.70	6.9	15.81	13.06
13	16.54	16.27	6.9	16.37	13.50
14	17.08	16.83	6.9	16.92	13.90
15	17.62	17.37	7.0	17.45	14.28
16	18.14	17.90	7.1	17.96	14.63
17	18.65	18.41	7.2	18.45	14.97
18	19.15	18.90	7.3	18.92	15.29
19	19.64	19.38	7.4	19.37	15.60
20	20.11	19.84	7.5	19.81	15.89
21	20.72	20.60	6.6	20.56	16.72
22	21.29	21.24	6.4	21.19	17.22
23	21.84	21.82	6.4	21.74	17.62
24	22.36	22.35	6.4	22.26	17.98
25	22.87	22.86	6.4	22.75	18.31
26	23.36	23.34	6.5	23.21	18.62
27	23.83	23.80	6.5	23.65	18.91
28	24.29	24.25	6.6	24.07	19.19
29	24.73	24.68	6.7	24.47	19.45
30	25.15	25.09	6.8	24.86	19.71

Appendix 2: Calculated drawdown for cost minimization

Simulation time(yr)	Calculated drawdown(m)				
	Akaki	Dalota	Fanta	Dukem up	Dukem down
1	2.45	2.65	4.00	3.01	3.52
2	4.35	4.65	5.03	4.96	5.12
3	6.05	6.35	5.72	6.55	6.29
4	7.61	7.87	6.26	7.95	7.26
5	9.05	9.25	6.73	9.22	8.10
6	10.40	10.53	7.15	10.40	8.87
7	11.67	11.73	7.53	11.50	9.58
8	12.86	12.86	7.88	12.54	10.23
9	14.00	13.92	8.21	13.52	10.85
10	15.07	14.94	8.51	14.46	11.44
11	15.47	15.40	7.02	14.99	12.20
12	15.99	15.94	6.88	15.55	12.71
13	16.52	16.49	6.89	16.10	13.15
14	17.05	17.03	6.95	16.63	13.55
15	17.58	17.56	7.03	17.15	13.92
16	18.09	18.08	7.12	17.65	14.27
17	18.60	18.58	7.22	18.13	14.60
18	19.09	19.06	7.33	18.59	14.92
19	19.57	19.53	7.44	19.04	15.22
20	20.03	19.99	7.55	19.47	15.51
21	20.68	20.68	6.58	20.19	16.27
22	21.27	21.29	6.42	20.78	16.73
23	21.82	21.84	6.38	21.31	17.11
24	22.36	22.37	6.39	21.81	17.46
25	22.87	22.87	6.42	22.29	17.78
26	23.36	23.35	6.47	22.74	18.08
27	23.83	23.81	6.54	23.17	18.37
28	24.29	24.25	6.61	23.59	18.64
29	24.73	24.67	6.68	23.99	18.90
30	25.15	25.08	6.76	24.37	19.15

Appendix 3: Calculation of pipe installation and well drilling cost

Wells	Distance from reservoir(m)	Pipe cost/ unit meter ETB	Pipe Installation cost(ETB)	Well drilling cost(ETB)	Total cost(ETB)
Akaki_276	2980	266	793918	496591	1290509
Akaki_277	3269	266	870876	496591	1367467
Akaki_278	3638	266	969275	496591	1465866
Akaki_279	3440	266	916496	496591	1413087
Akaki_282	3866	266	1029848	496591	1526439
Akaki_284	4162	266	1108811	496591	1605402
Akaki_285	4327	266	1152613	496591	1649204
Akaki_286	4496	266	1197813	496591	1694404
Akaki_287	4597	266	1224523	496591	1721114
Akaki_290	4834	266	1287816	496591	1784407
Akaki_291	5099	266	1358425	496591	1855016
fanta_2	5622	266	1497809	496591	1994400
fanta_3	5616	266	1496004	496591	1992595
fanta_4	5823	266	1551229	496591	2047820
fanta_5	6028	266	1605782	496591	2102373
fanta_6	6301	266	1678651	496591	2175242
Dal_1	869	266	231489	496591	728080
Dal_2	1154	266	307433	496591	804024
Dal_3	1552	266	413434	496591	910025
Dal_4	2114	266	563130	496591	1059721
Dal_5	2668	266	710698	496591	1207289
Dal_6	3226	266	859481	496591	1356072
Dup_1	5335	266	1421360	496591	1917951
Dup_2	4893	266	1303602	496591	1800193
Dup_3	5484	266	1461011	496591	1957602
Dup_4	5153	266	1372820	496591	1869412
Dup_5	6100	266	1625022	496591	2121613
Dup_6	5804	266	1546215	496591	2042806
Dup_7	6467	266	1722757	496591	2219348
Dup_8	5583	266	1487223	496591	1983814
Ddwn_1	5936	266	1581336	496591	2077927
Ddwn_2	6369	266	1696671	496591	2193262
Ddwn_3	6811	266	1814397	496591	2310988
Ddwn_4	6846	266	1823731	496591	2320322
Ddwn_5	6691	266	1782439	496591	2279030
Ddwn_6	6910	266	1840779	496591	2337370
Total cost(ETB)			46,814,631	183,73,868	65,188,499